IMPROVEMENT OF A GLOBAL SPECTRAL MODEL BY INTRODUCING THE SECOND LAW OF THERMODYNAMICS*

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ABSTRACT

The second law of thermodynamics has never been taken into account in the traditional hydrodynamics and numerical weather prediction models, which is a serious oversight in the history of mechanics. Introducing the thermodynamic irreversibility into the hydrodynamic systems, the theory and method proposed in this study would not only lead the outputs of a numerical weather prediction model to noticeable improvement, but lead the structure of hydrodynamics to deepgoing transformation.

Key words: the second law of thermodynamics, numerical weather prediction, hydrodynamics

I. INTRODUCTION

The physical base on which the numerical weather prediction models are built is the classical hydrodynamics, which is, in nature, a product of the so–called projective operator technique (Hao and Yu 1981). In terms of statistical physics a fluid such as the atmosphere is a kind of many–body systems which have a vast amount of degree of freedom. It is impossible and unnecessary to reveal the character of a many–body system through describing the behaviors of every particle in the system. The traditional strategy is to project it onto a subspace spanned by the average pressure, temperature, density, velocity, etc. Provided we would have the physical laws (actually the mathematical expressions or equations corresponding to the laws) whose number is the same as that of the independent variables the corresponding dynamical system (described by a close system of equations then) would be solvable. So has just done the classical hydrodynamics where the three conservation laws on mass, energy and (3–dimensional) momentum and the diagnostic experimental law of the state equation for ideal gases have been employed. As a result, traditionally it is described by projecting just onto (for e.g. the dry and clean atmosphere) a subspace of 6–dimensions. Without doubt, the higher the dimensions of the projective subspace are, the more accurate description of the dynamic system would be reached. Extending the projective subspace or increasing the dimensions of the subspace by introducing more physical laws is therefore a direction toward enhancing the quality and accuracy of hydrodynamics and numerical weather prediction models. The first attempt along this direction is to introduce the second law of thermodynamics or entropy law.

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For a long time, the second law of thermodynamics that controls the spontaneous evolution direction of many–body systems has been paid no attention to, which is because, on one hand, this law is expressed in the form of inequality and, on the other hand, the deterministic world outlook that occupied a dominant position during the three centuries after the establishment of Newtonian mechanics had also deeply affected the fluid mechanics and atmospheric dynamics: in those fields of the many–body systems studies, it has not been taken enough notice of that such formulas of Newtonian mechanics as the Euler equation for an ideal fluid and the Navier–Stokes one for a viscous flow both could be derived from the Boltzmann equation for a distribution function that describes, in nature, statistical laws. In addition, although the classical law of energy conservation could be expressed in terms of the first law of thermodynamics, its special case had already been through studied within the Newtonian mechanics. Introducing the second law of thermodynamics into the dynamical system must therefore involve the problem of amendment of the classical determinism, which would be really solved only if the thermodynamic irreversibility is introduced into mechanics and thus the deepgoing transformation of the structure of mechanics takes place; this study in which the second law of thermodynamics has been introduced into the Global Spectral Model of the Institute of Atmospheric Physics, Chinese Academy of Sciences (Ji et al. 1989) to extend the projective subspace is just a kind of efforts to transform the structure of mechanics. The outputs of the extended model improved dramatically the forecasting results of the original model (for example, the 5th day forecasting rate of accuracy in terms of the anomaly correlation coefficients has been raised by the average of 9.7 %). In view of the universality of this strategy, it is expected that all the well–known models of hydrodynamics and numerical weather prediction in the world could be improved according to this strategy one by one.

II. PRINCIPLES

The standard mathematical expression for the classical second law of thermodynamics for an open system is as follows:

$$\frac{\mathrm{d}s}{\mathrm{d}t} \geqslant \frac{Q}{T_e}, \tag{1}$$

where s is the entropy per unit mass and the specific entropy for an ideal gas under the hypothesis of the local equilibrium could be determined by the temperature T and pressure P (Liu 1988); Q the heating rate and T_e the environmental temperature. It is obvious that Eq. (1) could be rewritten as the following equality:

$$\frac{\mathrm{d}s}{\mathrm{d}t} = \frac{Q}{T_e} + P, \tag{2}$$

where the term P should be definitely positive according to the second law of thermodynamics; T_e could be derived directly from the average over the T of the surrounding grid points. As a result, only the P is the newly–introduced independent variable in Eq. (2) and thus the problem is solvable. If the solved P is definitely positive at a certain grid point, which meets the requirement, no treatment is needed. In case the solved P is negative, which runs counter to the second law of thermodynamics, the simplest scheme of correction is to set $P = 0$, as done in this work. On the other hand, combining the first law of thermodynamics with the second law (Eq. (2)), we can easily have

$$P = \frac{T_e - T}{TT_e} Q. \tag{3}$$

It could be seen from Eq. (3) that the negative P corresponds to the sign of $T_e - T$ contrary to that of Q, and the correction of setting $P = 0$ in the case of $T_e \neq T$ is equivalent to the adiabatic treatment, i.e. setting $Q = 0$, which permits then refinding the new prognostic values of temperature through Eq. (2) so that the new prognostic outputs that obey the second law of thermodynamics can be worked out on their feedback to the whole system of model.

III. RESULTS AND DISCUSSION

In view of that the constraint of the second law of thermodynamics is substantially embodied in the definite positivity of the term P in its mathematical expression (2), which, in nature, exerts a certain restraint on the diabatic heating patterns Q that are traditionally expressed in light of parameterization, the model outputs with the different diabatic heating schemes would respond to the constraint of the second law of thermodynamics in different ways; the difference between the two kinds of heating schemes mentioned below lies in the restriction of $\zeta > 10^{-5}$ for the first heating scheme where ζ is the relative vorticity at the top of the boundary layer within the parameterizations of the large–scale precipitation and cumulus convection (Kuo–scheme) while no such restriction for the second scheme.

Next the overall efficiency of improvement by this method would be examined in terms of the anomaly correlation coefficients and root mean square (RMS) errors, and then the details of improvement of the patterns or features described through contrasting the concrete fields of heights.

Figure 1 shows the evaluation of improvement in the case of 21 waves and 9 levels (simply T21L9) with the first heating scheme; the sets of curves are the averages over the tests of the

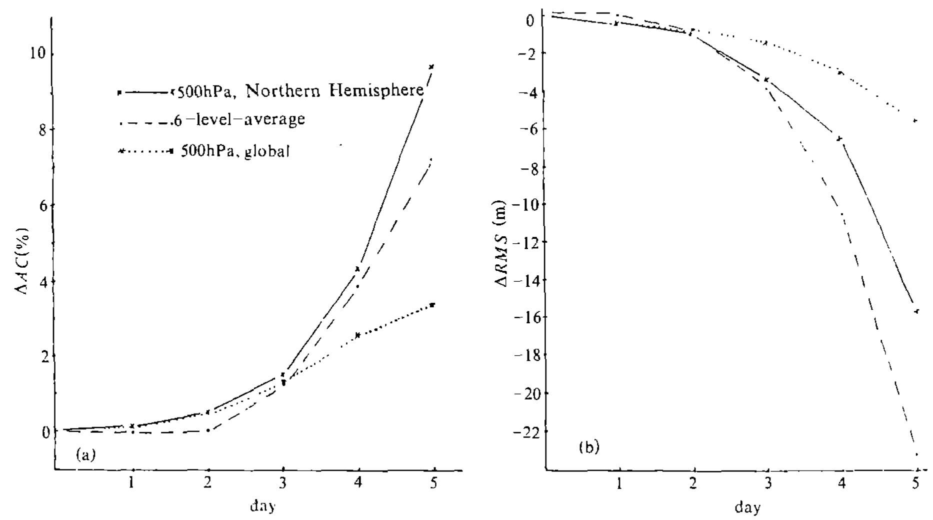


Fig. 1. The improvement in the evaluation for the outputs of the height fields in the case of T21L9 with the first heating scheme on the introduction of the second law of thermodynamics. (a) The anomaly correlation coefficients difference $\triangle AC$ between those with the second law of thermodynamics introduced and those without it; (b) the root–mean–square errors difference $\triangle RMS$.

forecasted height fields with the initial fields being the days of January 23, April 7, June 14 and November 5, 1979 which are representative of 4 seasons, respectively. It is seen from Fig.1 that the average increment of 9.7 % (even 15.3 % for the individual example of June 14) in the anomaly correlation coefficients for the 5th day forecasted 500 hPa fields of height has been reached by introducing the second law of thermodynamics.

For the same examples and the same resolution (T21L9) but with the second heating scheme (Fig. 2) the *RMS* error of the heights decreased by (e. g.) 37.2 m for the Northern Hemisphere 6–level–average (with the maximum decrement of 44.4 m for June 14), which is much larger than the 23.2 m with the first scheme, although the anomaly correlation coefficient with the second scheme increased by less than 4.0 % (e.g. only 3.4 % for the global 500 hPa fields of height).

In addition, the contrast experiments on the effect of the model resolution have also been done and it is found out that the case of T42L9 itself has already enhanced (e.g.) the Northern

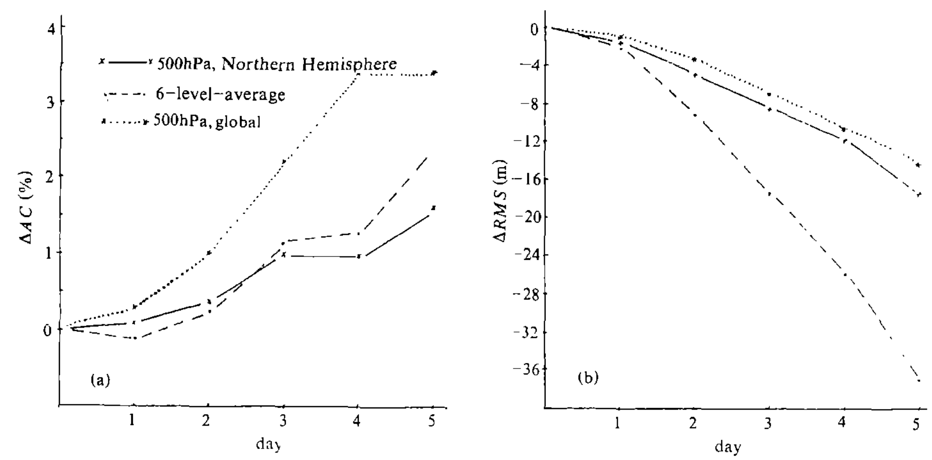


Fig. 2. As in Fig.1 but for the second heating scheme.

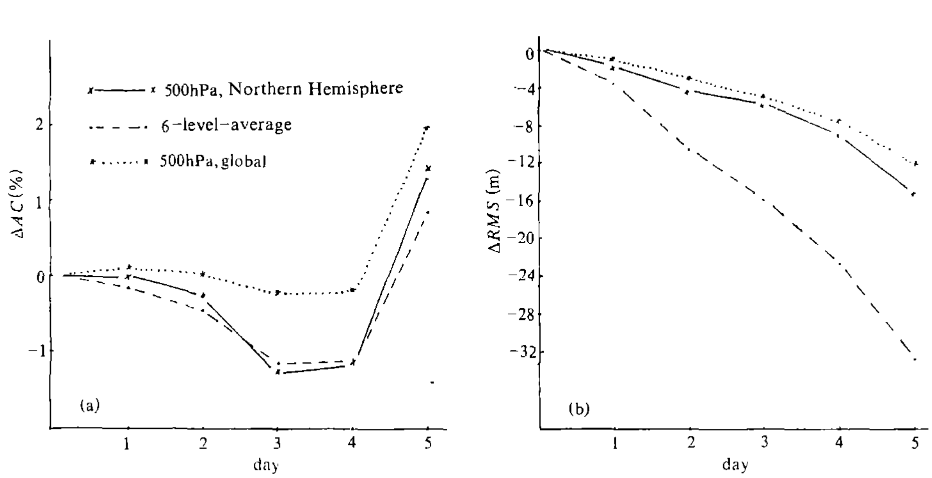
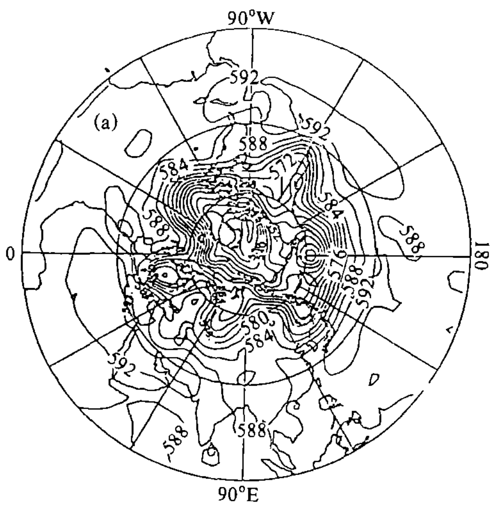


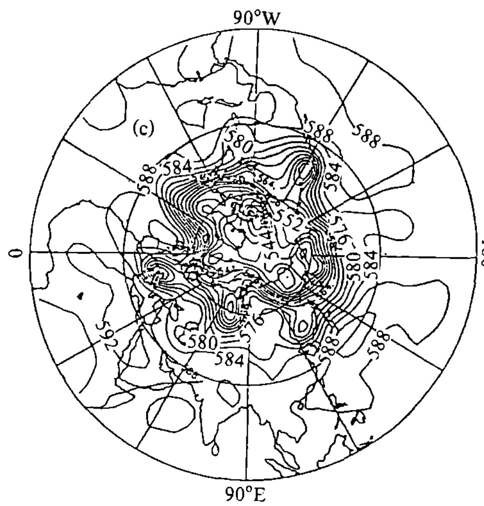
Fig. 3. As in Fig.2 but for the case of T42L9.

Hemisphere 6–level–average 500 hPa anomaly correlation coefficient to 63.6 % (Fig.3) from 42.2 % in the case of T21L9 before introducing the second law of thermodynamics. There seems little room for the improvement of the correlation coefficients (in fact improved by only 0.9 %, and the maximum being 2.0 % for the global 500 hPa correlation coefficient as showed in Fig.3); even so, the *RMS* errors of T42L9 have been averagely decreased (still taking the Northern Hemisphere 6–level–average as an example) by 32.2 m on introducing the restraint of the second law of thermodynamics, and the example of June 14 is of the maximum decrement of 42.2 m, which is very close to that of T21L9.

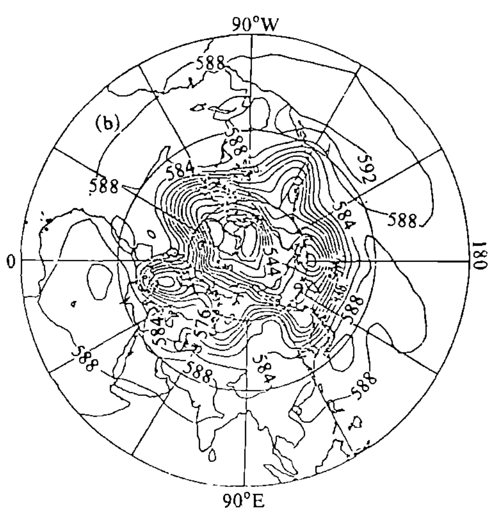
We will illuminate the concrete improvement of (500 hPa) height fields with the example of an intermediate improvement margin that is the case of T42L9 with the second heating scheme for the 3th day outputs of June 14, 1979 (as the results showed, the improvement in the case of T21 is better than in the T42, with the first heating scheme is better than with the second one,



MAX = 595.6 MIN = 526.6



MAX = 0.5955613403E+03 MIN = 0.5218132324E+03



MAX = 593.7 MIN = 525.2

Fig. 4. The comparison of the 3th day outputs of height fields of 500 hPa in the case of T42L9 with the second heating scheme employed and June 14, 1979 as the initial with the observations of June 17. (a) The outputs of the original model; (b) the outputs with the second law of thermodynamics introduced; (c) the observations on 17 June 1979.

and, there is a general trend of getting better with time, e.g. the improvement margin of the 5th day outputs is larger than the 3th). The observations of the 3th day (June 17, Fig.4c) show that there exist (e. g.) two longwave ridges (near 115°E and 45°E, respectively) and a large trough with a closed center between them over the Eurasian. It is easily seen that such qualitative character has been depicted by the model outputs by introducing the second law of thermodynamics (see Fig. 4b) while a "contrary " one by the original model outputs: a closed center appears in the two ridges, respectively, while it disappears in the trough (see Fig. 4a). In addition, the improvement in the depiction of the western Pacific subtropic high is also noticeable on introducing the second law of thermodynamics: the subtropical cell portrayed by the contours of 588 and 592 is close on the observations in either strength or position while in the outputs of the original model with no introduction of the second law of thermodynamics, not only the central range defined by the contour of 592 is very different from the observations, but the morphology of the subtropical high is greatly distorted compared with the observations (see Fig.4).

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