ON THE SYMMETRIC DEVELOPMENT OF MESOSCALE DISTURBANCES

Sun Litan (孙立潭) *and Zhao Ruixing* (赵瑞星)

Institute of Atmospheric Physics, Academia Sinica, Beijing

Received July 10, 1990

ABSTRACT

The effects of baroclinic basic flow on a paralleled mesoscale disturbance development are investigated. By using the WKB method, two-dimensional perturbation equations with the anelastic approximation are analyzed. The result indicates that the symmetric development of a mesoscale disturbance is due to the inhomogeneous thermal wind deviations and unstationality of the basic field.

Key words: baroclinic atmosphere , wave packet, symmetric development, mesoscale disturbance, thermal wind deviation

I. INTRODUCTION

Severe convective disturbances usually occur in the form of organized mesoscale systems such as squall line, mesoscale convective complex (MCC), etc. These organized mesoscale systems often take place in a distinct synoptic environment, playing very important roles not only in triggering and organizing smaller convection but also in participating in the feedback and interactions between the large, meso, and small scale systems. Therefore it is of great importance to investigate the mechanism of mesoscale system formation. Based on this point, we have studied the symmetric development of mesoscale disturbance under the circumstance of weak imbalance between winds and thermal fields.

Great progresses have been made in the past decades of mesoscale dynamics research, especially in the stability problems of ageostrophic paralleled disturbances under geostrophic basic flows. Kuo (1954) studied the symmetric instability (of disturbance). Ooyama (1966) used the symmetric instability theory to investigate the axisymmetric disturbances of typhoon. Hoskins (1974), Bennetts and Hoskins (1979), Emanuel (1979), and Ogura, et al. (1982) considered that mesoscale symmetric instability may play an important role in triggering and organizing the band-like convective activities. Recently, Kuo and seitter (1985) have made research on the instability of shearing geostrophic currents in neutral and partly unstable atmospheres. Zhang (1988) have discussed the symmetric instability problem in a bounded domain. In this paper, other than the works above, we will use the WKB method to investigate the evolution of an inertia-gravity wave packet superimposed on a weakly unbalanced thermal wind current.

II. GOVERNING EQUATION

Zhang (1980) pointed out the anelastic approximation model is suitable to discribe mesoscale motion. In the f-plane, the linearized anelastic model for the disturbances in baroclinic, shearing current can be written as

$$\begin{cases}\left(\dfrac{\partial}{\partial t}+\bar{u}\dfrac{\partial}{\partial x}\right)u+w\bar{u}_z-f_a v+\dfrac{\partial p'}{\partial x}=0,\\[2ex] \left(\dfrac{\partial}{\partial t}+\bar{u}\dfrac{\partial}{\partial x}\right)v+fu+\dfrac{\partial p'}{\partial y}=0,\\[2ex] \left(\dfrac{\partial}{\partial t}+\bar{u}\dfrac{\partial}{\partial x}\right)w+\dfrac{\partial p'}{\partial z}-\theta=0,\\[2ex] \left(\dfrac{\partial}{\partial t}+\bar{u}\dfrac{\partial}{\partial x}\right)\theta-M^2v+N^2w=Q,\\[2ex] \dfrac{\partial u}{\partial x}+\dfrac{\partial v}{\partial y}+\dfrac{\partial w}{\partial z}=0,\end{cases} \tag{1}$$

where

$$(u,v,w,\theta)\equiv\rho\left(u',v',w',\frac{g}{\bar{\theta}}\theta'\right),\ M^2=-\frac{g}{\bar{\theta}}\frac{\partial\bar{\theta}}{\partial y},\ N^2=\frac{g}{\bar{\theta}}\frac{\partial\bar{\theta}}{\partial z},\ Q=\frac{\bar{\rho}g}{c_pT}\dot{H},\ f_a=f-\bar{u}_y,$$

and, bar represents basic fields; prime the disturbed fields; $\bar{\rho}\dot{H}$ the heating rate of unit volume, $\bar{u}(y,z)$ basic flow; f_a the absolute vorticity of basic flow, $\bar{\theta}$ basic potential temperature.

For the sake of mathematical treatment, we consider the symmetric development of disturbances and, let $\partial F/\partial x=0$ where F is an arbitrary quantity. Then, introducing the streamfunction ψ in (y,z) plane, we have

$$\begin{cases}v=\dfrac{\partial\psi}{\partial z},\\[2ex] w=-\dfrac{\partial\psi}{\partial y},\end{cases} \tag{2}$$

and equation set (1) can be rewritten as

$$\begin{cases}\dfrac{\partial\nabla^2\psi}{\partial t}=-f\dfrac{\partial u}{\partial z}-\dfrac{\partial\theta}{\partial y},\\[2ex] \dfrac{\partial u}{\partial t}=f_a\dfrac{\partial\psi}{\partial z}+\bar{u}_z\dfrac{\partial\psi}{\partial y},\\[2ex] \dfrac{\partial\theta}{\partial t}=M^2\dfrac{\partial\psi}{\partial z}+N^2\dfrac{\partial\psi}{\partial y},\end{cases} \tag{3}$$

where, we let $Q=0$, i.e., having no consideration of heating sources. Eliminating the variables u and θ, we obtain the single equation of $\psi(y,z)$

$$\nabla^2\psi_{tt}+ff_a\psi_{zz}+(f\bar{u}_z+M^2)\psi_{yz}+N^2\psi_{yy}+\left(\frac{\partial ff_a}{\partial z}+\frac{\partial M^2}{\partial y}\right)\frac{\partial\psi}{\partial z}+\left(\frac{\partial f\bar{u}_z}{\partial z}+\frac{\partial N^2}{\partial y}\right)\frac{\partial\psi}{\partial y}=0. \tag{4}$$

Ooyama (1966), Hoskins (1974) and Zhang (1988) discussed the eigenvalue problems of Eq. (4) under thermal wind balance, i.e. the last two terms were equal to zero. In this paper, we use WKB method to study the complete form of Eq. (4).

III. WAVE GROUP ANALYSIS

Let us introduce the "stretched" space coordinates $Y=\varepsilon y$, $Z=\varepsilon z$, and the "slowly varying time coordinate $T=\varepsilon t$, and suppose the streamfunction has the following form

$$\psi=A\exp(i\tau/\varepsilon), \tag{5}$$

where

$$A=A(Y,Z,T)=A_0(Y,Z,T)+\varepsilon A_1(Y,Z,T)+\cdots, \tag{6}$$

$$\tau=kY+nZ-\omega T, \tag{7}$$

ε is a small parameter, and τ represents the phase of gravity wave.

Substituting Eqs. (5), (6) and (7) into Eq. (4), rewriting Eq. (4) in the form of the power series of ε, and letting the coefficients before every power of ε be zero, we have

$$\varepsilon^0: \quad \omega^2\nu^2=n^2ff_a+nk(f\bar{u}_z+M^2)+k^2N^2, \tag{8}$$

$$\begin{aligned}\varepsilon^1: \quad & 2\omega\frac{\partial}{\partial T}(\nu^2A_0)+\frac{\partial A_0}{\partial Y}[-2\omega^2k+n(f\bar{u}_z+M^2)+2kN^2] \\ & +\frac{\partial A_0}{\partial z}[-2\omega^2n+k(f\bar{u}_z+M^2)+2nff_a]+\nu^2\frac{\partial\omega}{\partial T}A_0 \\ & -\omega^2A_0\left(\frac{\partial k}{\partial Y}+\frac{\partial n}{\partial Z}\right)+ff_a\frac{\partial n}{\partial Z}A_0+(f\bar{u}_z+M^2)\frac{\partial n}{\partial Y}A_0 \\ & +N^2\frac{\partial k}{\partial Y}A_0+\left(\frac{\partial ff_a}{\partial Z}+\frac{\partial M^2}{\partial Y}\right)nA_0+\left(\frac{\partial f\bar{u}_z}{\partial Z}+\frac{\partial N^2}{\partial Y}\right)kA_0=0;\end{aligned} \tag{9}$$

where $\nu^2=k^2+n^2$. Eq. (8) is the dispersive equation and Eq. (9) the amplitude equation.

Using the dispersive relation of Eq. (8), we can easily obtain the group velocities along the Y and Z axes respectively

$$C_{gY}=\frac{\partial\omega}{\partial k}=\frac{\nu^2}{2\omega}[-2\omega^2k+n(f\bar{u}_z+M^2)+2kN^2], \tag{10}$$

$$C_{gZ}=\frac{\partial\omega}{\partial n}=\frac{\nu^2}{2\omega}[-2\omega^2n+k(f\bar{u}_z+M^2)+2nff_a]. \tag{11}$$

Making use of the notation

$$\frac{d_g}{dt}=\frac{\partial}{\partial T}+C_{gY}\frac{\partial}{\partial Y}+C_{gZ}\frac{\partial}{\partial Z}, \tag{12}$$

and Eq. (8), we can also get the following equations governing the disturbed wave parameters:

$$\frac{d_gk}{dT}=-\frac{\nu^2}{2\omega}\left(n^2\frac{\partial ff_a}{\partial Y}+nk\frac{\partial(f\bar{u}_z+M^2)}{\partial Y}+k^2\frac{\partial N^2}{\partial Y}\right), \tag{13}$$

$$\frac{d_gn}{dT}=-\frac{\nu^2}{2\omega}\left(n^2\frac{\partial ff_a}{\partial Z}+nk\frac{\partial(f\bar{u}_z+M^2)}{\partial Z}+k^2\frac{\partial N^2}{\partial Z}\right), \tag{14}$$

$$\frac{d_g\nu^2}{dT}=-\frac{\nu^2}{\omega}\left(n^2\frac{\partial ff_a}{\partial l_0}+nk\frac{\partial(f\bar{u}_z+M^2)}{\partial l_0}+k^2\frac{\partial N^2}{\partial l_0}\right), \tag{15}$$

$$\frac{d_g(k/n)}{dT}=-\frac{\nu^2}{2\omega n^2}\left(n^2\frac{\partial ff_a}{\partial l_\perp^0}+nk\frac{\partial(f\bar{u}_z+M^2)}{\partial l_\perp^0}+k^2\frac{\partial N^2}{\partial l_\perp^0}\right), \tag{16}$$

where $\boldsymbol{l}_0=\nu^2(k\boldsymbol{j}+n\boldsymbol{k})$, $\boldsymbol{l}_\perp^0=\nu^2(n\boldsymbol{j}-k\boldsymbol{k})$. Substituting Eqs. (10) and (11) into (9) and multiplying the both sides of Eq. (9) by A_0, we obtain the energy equation

$$\frac{\partial(\nu^2 A_0^2)}{\partial T}+\nabla\cdot(\nu^2 A_0^2 C_g)-\frac{A_0^2}{\omega}\omega\nabla\cdot(\nu^2 C_g)+\frac{A_0^2}{\omega}\Big[\nu^2\frac{\partial\omega}{\partial T}+\omega\frac{\partial\nu^2}{\partial T}-\omega^2\Big(\frac{\partial k}{\partial Y}+\frac{\partial n}{\partial Z}\Big)$$

$$+ff_a\frac{\partial n}{\partial Z}+(f\bar{u}_z+M^2)\frac{\partial n}{\partial Y}+N^2\frac{\partial k}{\partial Y}+\Big(\frac{\partial ff_a}{\partial Z}+\frac{\partial M^2}{\partial Y}\Big)n+\Big(\frac{\partial f\bar{u}_z}{\partial Z}+\frac{\partial N^2}{\partial Y}\Big)k\Big]=0. \tag{17}$$

Using Eqs. (10) and (11), we obtain

$$\omega\nabla\cdot(\nu^2 C_g)=-\omega\Big[\omega\Big(\frac{\partial k}{\partial Y}+\frac{\partial n}{\partial Z}\Big)+k\frac{\partial\omega}{\partial Y}+n\frac{\partial\omega}{\partial Z}\Big]$$

$$+\omega\Big[\frac{\partial k}{\partial Y}\frac{N^2}{\omega}-\frac{kN^2}{\omega^2}\frac{\partial\omega}{\partial Y}+\frac{k}{\omega}\frac{\partial N^2}{\partial Y}\Big]+\omega\Big[\frac{ff_a}{\omega}\frac{\partial n}{\partial Z}-\frac{nff_a}{\omega^2}\frac{\partial\omega}{\partial Z}+\frac{n}{\omega}\frac{\partial ff_a}{\partial Z}\Big]$$

$$+\omega\Big[\frac{1}{2\omega}(f\bar{u}_z+M^2)\Big(\frac{\partial n}{\partial Y}+\frac{\partial k}{\partial Z}\Big)-\frac{(f\bar{u}_z+M^2)}{2\omega^2}\Big(n\frac{\partial\omega}{\partial Y}+k\frac{\partial\omega}{\partial Z}\Big)\Big]$$

$$+\frac{\omega}{2\omega}\Big[n\frac{\partial(f\bar{u}_z+M^2)}{\partial Y}+k\frac{\partial(f\bar{u}_z+M^2)}{\partial Z}\Big], \tag{18}$$

and from (8), we have

$$\nu^2\frac{\partial\omega}{\partial T}=-\frac{\omega^2}{2\omega}\frac{\partial\nu^2}{\partial T}+\frac{2nff_a}{2\omega}\Big(-\frac{\partial\omega}{\partial Z}\Big)-\frac{\Big(n\frac{\partial\omega}{\partial Y}+k\frac{\partial\omega}{\partial Z}\Big)}{2\omega}(f\bar{u}_z+M^2)$$

$$+\frac{2k}{2\omega}\Big(-\frac{\partial\omega}{\partial Z}\Big)N^2+\frac{1}{2\omega}\Big(\frac{\partial ff_a}{\partial T}n^2+nk\frac{\partial}{\partial T}(f\bar{u}_z+M^2)+k^2\frac{\partial N^2}{\partial T}\Big). \tag{19}$$

Substitute Eqs. (18) and (19) into (17), and make use of the following kinetic relationship of wave parameters:

$$\frac{\partial\omega}{\partial Y}=-\frac{\partial k}{\partial T},\frac{\partial\omega}{\partial Z}=-\frac{\partial n}{\partial T},\frac{\partial n}{\partial Y}=\frac{\partial k}{\partial Z}. \tag{20}$$

Then, Eq. (17) can be rewritten as

$$\frac{\partial(\nu^2 A_0^2)}{\partial T}+\nabla\cdot(\nu^2 A_0^2 C_g)+\frac{A_0^2}{2\omega}\Big[n\Big(\frac{\partial ff_a}{\partial Z}+\frac{\partial M^2}{\partial Y}\Big)$$

$$+k\Big(\frac{\partial f\bar{u}_z}{\partial Z}+\frac{\partial N^2}{\partial Y}\Big)\Big]+\frac{A_0^2}{2\omega^2}\Big[k^2\frac{\partial N^2}{\partial T}+nk\frac{\partial(f\bar{u}_z+M^2)}{\partial T}+n^2\frac{\partial ff_a}{\partial T}\Big]=0. \tag{21}$$

Integrating Eq. (21) in the wave packet domain, and supposing the wave packet has zero amplitude at its margine, i.e., $A_0=0$, we obtain the disturbed wave packet energy equation

$$\frac{\partial}{\partial T}\iint_\sigma(\nu^2 A_0^2)\,dY\,dZ=-\iint_\sigma\frac{A_0^2}{2\omega}\Big[n\Big(\frac{\partial ff_a}{\partial Z}+\frac{\partial M^2}{\partial Y}\Big)+k\Big(\frac{\partial f\bar{u}_z}{\partial Z}+\frac{\partial N^2}{\partial Y}\Big)\Big]dY\,dZ$$

$$-\iint_\sigma\frac{A_0^2}{2\omega^2}\Big[k^2\frac{\partial N^2}{\partial T}+nk\frac{\partial(f\bar{u}_z+M^2)}{\partial T}+n^2\frac{\partial ff_a}{\partial T}\Big]dY\,dZ, \tag{22}$$

where σ represents whole region of the packet.

IV. WAVE PACKET SYMMETRIC DEVELOPMENT

According to the wave packet energy $E=\iint_\sigma \nu^2 A_0^2\, dY\, dZ$, we can define the criteria for the development of wave packet in (Y, Z) plane as

$$\frac{\partial E}{\partial T}=\frac{\partial}{\partial T}\iint_\sigma \nu^2 A_0^2\, dY dZ \begin{cases} >0, & \text{for symmetric developing} \\ <0. & \text{for symmetric decaying} \end{cases} \tag{23}$$

(1) If the basic fields satisfy the thermal wind balance, then

$$M^2=-g\frac{\partial \ln\bar{\theta}}{\partial y}=f\bar{u}_z=f\bar{u}_\theta \tag{24}$$

and

$$\frac{\partial f f_a}{\partial Z}=\frac{\partial}{\partial Z}[f(f-\bar{u}_y)]=-f\frac{\partial \bar{u}_z}{\partial Y}=-\frac{\partial M^2}{\partial Y}, \tag{25}$$

$$\frac{\partial f\bar{u}_z}{\partial Z}=\frac{\partial M^2}{\partial Z}=-g\frac{\partial^2 \ln\bar{\theta}}{\partial Z\partial Y}=-\frac{\partial N^2}{\partial Y}. \tag{26}$$

Substituting Eqs. (25) and (26) into (22), we obtain

$$\frac{\partial}{\partial T}\iint_\sigma \nu^2 A_0^2\, dY dZ=-\iint_\sigma \frac{A_0^2}{2\omega^2}\left[k^2\frac{\partial N^2}{\partial T}+2nk\frac{\partial M^2}{\partial T}+n^2\frac{\partial f f_a}{\partial T}\right]dY\, dZ. \tag{27}$$

Thus the following wave packet energy conservation theorem can be reached: If the large-scale basic field is stationary and thermal wind-balanced, then the symmetric wave packet energy is conserved, and in such case the packet is neutral.

(2) If the basic fields are thermal wind-balanced but unstationary, then the necessary condition for a wave packet to develop is

$$k^2\frac{\partial N^2}{\partial T}+2nk\frac{\partial M^2}{\partial T}+n^2\frac{\partial f f_a}{\partial T}<0. \tag{28}$$

If $\partial N^2/\partial T>0$, the inequality can be rewritten as

$$\left(k+n\frac{\partial M^2/\partial T}{\partial N^2/\partial T}\right)^2+\left[\frac{\partial f f_a/\partial T}{\partial N^2/\partial T}-\left(\frac{\partial M^2/\partial T}{\partial N^2/\partial T}\right)^2\right]n^2<0; \tag{28'}$$

so we have the necessary condition for a packet development when $\partial N^2/\partial T>0$

$$\left(\frac{\partial M^2}{\partial T}\right)^2-\frac{\partial f f_a}{\partial T}\frac{\partial N^2}{\partial T}>0. \tag{29}$$

Similarly, we can also get the sufficient condition for a wave packet to develop when $\partial N^2/\partial T<0$ as follows:

$$\left(\frac{\partial M^2}{\partial T}\right)^2-\frac{\partial f f_a}{\partial T}\frac{\partial N^2}{\partial T}<0. \tag{30}$$

(3) If the basic field is stationary, but there is no balance in the thermal field and the wind field, that is, there exists a thermal wind deviation, the development of wave packet will be discussed as follows:

Denoting the thermal wind by $\bar{u}_\theta$, and $M^2=-g\,\partial\ln\theta/\partial y=f\bar{u}_\theta$, and the thermal wind deviation by $\Delta\bar{u}_\theta=\bar{u}_\theta-\bar{u}_z$, we have

$$\frac{\partial E}{\partial T}=\iint_{\sigma}\frac{fA_0^2}{2\omega}\left[k\frac{\partial \Delta \bar{u}_\theta}{\partial Z}-n\frac{\partial \Delta \bar{u}_\theta}{\partial Y}\right]dY\,dZ=-\iint_{\sigma}\frac{fv^2A_0^2}{2\omega}[l_\perp^0\cdot\nabla(\Delta\bar{u}_\theta)]dY\,dZ, \qquad (31)$$

where, $l_\perp^0$ is defined as before, and we can easily prove

$$l_\perp^0//C_g/|C_g|. \qquad (32)$$

According to Eq. (31), we can analyze the disturbed wave packet development caused by the basic thermal wind deviation. The symmetric development of a disturbed wave packet is associated with the distribution of the thermal wind deviation in the packet domain. If the gradient direction of the thermal wind deviation field is not perpendicular to the group velocity C_g (i.e. the direction of wave packet energy propagation), the wave packet may develop; and if the gradient direction of the thermal wind deviation coincides with C_g, the packet will develop at maximum rate: and oppositely, the packet will decay at maximum rate when the gradient direction of the thermal wind deviation is opposite to C_g. So that we have the following models, as shown in Fig. 1.

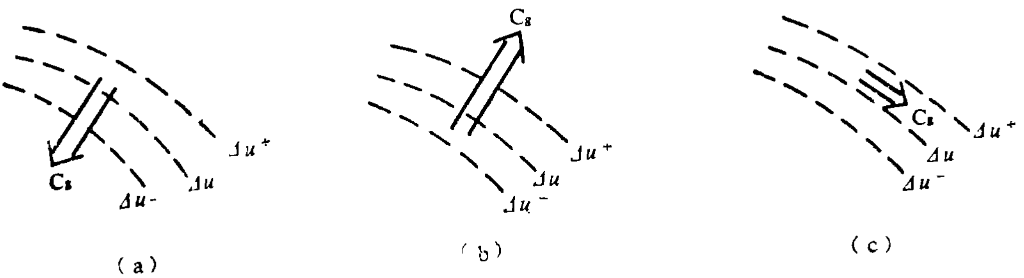

Fig. 1. Basic modes of the symmetric development of mesoscale disturbances: (a) for developing; (b) for decaying; and (c) for neutral.

Since the gravity waves propagate in bidirection when waves develop in one direction the waves in other direction must decay.

The developing or decaying of a wave packet is determined by the distribution of thermal wind deviation of basic fields. As a result, the developing or decaying of gravity wave packet is associated with the adjustment between the geostrophic wind and the potential temperature field. When developing, the disturbed wave packet gets energy from the basic field so as to make the thermal wind deviation and inhomogeneity vanish, finally the thermal wind balance will be established and the disturbances is no longer developing. The real atmosphere is aslo in such a dynamic equilibrium of both disturbances and basic fields.

V. CONCLUSIONS

The evolution of a gravity wave packet has been investigated by using the WKB and multiple scale method mentioned above, now it can be concluded as follows:

(1) If the basic fields are stationary and satisfy thermal wind balance, the disturbed wave packet energy is conserved.

(2) If the basic fields satisfy thermal wind balance but unstationary, then the necessary condition for development when $\partial N^2/\partial T>0$ is

$$\left(\frac{\partial M^2}{\partial T}\right)^2-\frac{\partial ff_a}{\partial T}\frac{\partial N^2}{\partial T}>0,$$

and the sufficient condition for development when $\partial N^2/\partial T<0$ is

$$\left(\frac{\partial M^2}{\partial T}\right)^2 - \frac{\partial f f_a}{\partial T}\frac{\partial N^2}{\partial T} < 0.$$

(3) If the basic fields are stationary, the symmetric developing or decaying of wave packets is associated with the distribution of thermal wind deviation of the basic field. The process of developing or decaying is an adjustment of thermal wind. Thus, when basic fields are unstationary the symmetric developing or decaying of a wave packet under the circumstance of thermal wind balance is associated with the evolution process of basic fields.

(4) The bidirectional propagating inertial-gravity wave packets will develop in one direction, but decay in the opposite if the basic fields are stationary, and these effects are strongest when the gradient direction of thermal wind deviation coincides with the propogating direction of the wave packets.

REFERENCES

Bennetts, D. A. and Hoskins, B. J. (1979), Conditional symmetric stability—a possible explanation for frontal rainbands, *Quart. J. R. Met. Soc.*, **105**: 945—966.

Chao Jiping (1980), The gravitational wave in non-uniform stratification atmosphere and its preliminary application for the prediction of heavy rainfall, *Scientia Atmospherica Sinica*, **4**: 230—235 (in Chinese).

Emanuel, K. E. (1979), Inertial instability and mesoscale convection system, Part I: Linear theory of inertial instability in rotating viscous fluids, *J. Atmos. Soc.*, **36**: 2425—2449.

Hoskins, B. J. (1974), The role of potential vorticity in symmetric stability and instability, *Quart. J. R. Met. Soc.*, **100**: 480—482.

Kuo, H. L. (1954), Symmetric disturbances in a thin layer of fluid subject to a horizontal temperature gradient and rotation, *J. Meteor.*, **11**: 399—411.

Kuo, H. L., and Seitter, K. L. (1985), Instability of shearing geostrophic currents in neutral and partly unstable atmospheres, *J. Atmos. Sci.*, **42**: 331—345.

Liu Shikuo et al. (1987) Wave action and stability of internal inertial gravity waves in geophysical fluid, *Scientia Atmospherica Sinica*, **11**: 12—22 (in Chinese).

Ogura, Y. et al. (1982), Possible triggering mechanisms for severe storms in SESAME AVE IV (9—10 May 1979), *Bull. Amer. Met. Soc.*, **63**: 503—515.

Ooyama, K. (1966), On the stability of baroclinic circular vortex: a sufficient criterion for instability, *J. Atmos. Sci.*, **23**: 43—53.

Zhang Kesu (1988), On the mesoscale instability of a baroclinic flow, I: symmetric instability, *Acta Meteor, Sinica*, **2**: 135—145.

Zhang Kesu (1980), A comparison study of several atmospheric dynamic models, *Scientia Sinica*, **23**: 277—287 (in Chinese).