INFLUENCES OF TOPOGRAPHY ON BAROCLINIC SOLITARY ROSSBY WAVES IN A MULTILEVEL MODEL

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ABSTRACT

The KdV equation with topography included in an N–level model is derived. It is shown that if the topography exists, the KdV equation may describe the solitary Rossby waves in the case of basic current without vertical shear, and it is no necessary to introduce the MKdV equation. The results of calculations show that the change of horizontal shear pattern of basic flow may cause an important change of the streamline pattern of the solitary waves with the odd meridional wavenumber m, and has no effect for the even meridional wavenumber m. The vertical shear increases the steepness of the barotropic solitary modes, and it has a complicated effect on the baroclinic modes. The influences of topographic slope on the solitary waves are very great. The southern and northern slopes of topography may cause different solitary wave patterns, with the effect of northern slope greater. The effect of Froude number on the solitary waves is generally to steepen the solitary waves, however, the effect also depends on the meridional wavenumber m and the modes of solitary wave.

Key words: topography, solitary wave, N–level model

I. INTRODUCTION

Lu (1987, 1988) has studied the influences of the horizontal shear of flow and topography on the streamline pattern of the solitary Rossby waves by using a barotropic model. Hukuda (1979) discussed the effects of vertical shear of basic current by using a two–level model. The barotropic model cannot describe the baroclinity of atmosphere, and the two–level model may be excessively simple for denoting the vertical shear of the flow, thus the real baroclinity of atmosphere cannot be expressed appropriately. However, the multilevel model may describe quite well the vertical structure of atmosphere. For this reason we will derive an N–level model including topography to extend Hukuda's work, and study the effects of vertical and horizontal shears of basic current and topography on the solitary Rossby waves. For simplicity, we will calculate the streamline patterns of solitary waves by using a three–level model.

II. KDV EQUATION IN AN N–LEVEL MODEL

Eliminating ω between the vorticity equation on level l and the thermo–dynamical equation on level $l+1/2$, and utilizing the boundary conditions $\omega_{1/2}=0$ at upper boundary, and at lower boundary

$$\omega_{N+1/2} = -\rho_N g\left(\frac{\partial\psi_N}{\partial x}\frac{\partial h}{\partial y} - \frac{\partial\psi_N}{\partial y}\frac{\partial h}{\partial x}\right),$$

and then introducing the scales

$$\psi = LV\psi', \quad t = \frac{L}{V}t', \quad \beta = \frac{V}{L^2}\beta', \quad (x,y) = L(x',y'), \quad h = Hh',$$

yield a set of potential vorticity equations

$$\begin{aligned}
&\frac{\partial}{\partial t'}\left[\nabla^2\psi_1' + F_r\left(\psi_2' - \psi_1'\right)\right] + J\left[\psi_1', \nabla^2\psi_1' + \beta' y' + F_r\left(\psi_2' - \psi_1'\right)\right] = 0,\\
&\frac{\partial}{\partial t'}\left[\nabla^2\psi_l' + F_r\left(\psi_{l+1}' - 2\psi_l' + \psi_{l-1}'\right)\right] + J\left[\psi_l', \nabla^2\psi_l' + \beta' y' + F_r\left(\psi_{l+1}' - 2\psi_l' + \psi_{l-1}'\right)\right] = 0, \qquad (1)\\
&\frac{\partial}{\partial t'}\left[\nabla^2\psi_N' - F_r\left(\psi_N' - \psi_{N-1}'\right)\right] + J\left[\psi_N', \nabla^2\psi_N' + \beta' y' - F_r\left(\psi_N' - \psi_{N-1}'\right) + H_1 h'\right] = 0,
\end{aligned}$$

$$(l = 2,3,\cdots,N-1)$$

where

$$\sigma = -\alpha\frac{\partial\ln\theta_s}{\partial p}, \qquad F_r = \frac{f^2L^2}{\sigma\triangle p^2}, \qquad H_1 = \frac{f\rho_N gL}{\triangle pV}H,$$

correspond to the static stability parameter, the rotating Froude number, and the effect factor of topography, respectively.

The boundary conditions may be written as

$$\frac{\partial\psi_l'}{\partial x'} = 0, \qquad y' = 0,1. \qquad (l = 1,2,\cdots,N) \qquad (2)$$

Similar to Clarke (1971), we consider the following transform

$$\psi_l' = \psi_l'(x' - ct', y'), \qquad (3)$$

then we may obtain the KdV equation independing of time and the permanent form of solitary wave which satisfies the KdV equation in the form (see Appendix for details)

$$C_1E_1\frac{d\Phi}{d\xi} + E_2\Phi\frac{d\Phi}{d\xi} + E_3\frac{d^3\Phi}{d\xi^3} = 0, \qquad (4)$$

where

$$\begin{aligned}
E_1 &= \int_0^1\sum_{l=1}^N\left[\Psi_l^2/(\bar{u}_l - C_0)\right]M_l(y)dy,\\
E_2 &= -\int_0^1\sum_{l=1}^N\left[\Psi_l^3/(\bar{u}_l - C_0)\right]\frac{d}{dy}M_l(y)dy,\\
E_3 &= \int_0^1\sum_{l=1}^N\Psi_l^2dy,\\
M_1(y) &= \left[\beta - \frac{d^2\bar{u}_1}{dy^2} + F_r(\bar{u}_1 - \bar{u}_2)\right]/(\bar{u}_1 - C_0), \qquad (5)\\
M_l(y) &= \left[\beta - \frac{d^2\bar{u}_l}{dy^2} - F_r(\bar{u}_{l-1} - 2\bar{u}_l + \bar{u}_{l+1})\right]/(\bar{u}_l - C_0),\\
M_N(y) &= \left[\beta - \frac{d^2\bar{u}_N}{dy^2} - F_r(\bar{u}_{N-1} - \bar{u}_N) + H_1\frac{dh}{dy}\right]/(\bar{u}_N - C_0).
\end{aligned}$$

$$(l = 2,3,\cdots,N-1)$$

The KdV equation (4) has a special solution ($\Phi \to 0$ when $\xi \to \pm\infty$) referred to as soliton, namely,

$$\Phi(\xi) = \mathrm{sgn}(E_2 E_3)\mathrm{sech}^2(\mu\xi), \tag{6}$$

where

$$C_1 = -\frac{E_2}{3E_1}\mathrm{sgn}(E_2 E_3),$$
$$\mu = \left|\frac{E_2}{12E_3}\right|^{1/2}, \tag{7}$$

and μ expresses the steepness of baroclinic solitary Rossby waves.

III. SOLITARY ROSSBY WAVES IN THE CASES OF PRESENCE OF FLOW WITH WEAK HORIZONTAL SHEAR AND TOPOGRAPHY

Assume the basic flow as

$$\bar{u}_l = \bar{u}_0 + S_l \bar{u}_s + \delta k y, \qquad (l = 1,2,\cdots,N) \tag{8}$$

where $S_l\,\bar{u}_s$ expresses the vertical shear of flow, and S_l is taken as different constants at different levels, $\bar{u}_0$, $\bar{u}_s$ and k are the specified constants, $\delta(\ll 1)$ is a small parameter denoting the intensity of horizontal shear of the basic flow.

Expand Ψ and C_0 in powers of δ as follows:

$$\begin{cases} \overline{\Psi}_l = \overline{\Psi}_l^{(0)} + \delta\overline{\Psi}_l^{(1)} + \delta^2\overline{\Psi}_l^{(2)} + \cdots, \\ C_0 = C_{00} + \delta C_{01} + \cdots. \end{cases} \tag{9}$$

For the sake of simplicity, take $\mathrm{d}h/\mathrm{d}y = l_1\ h_y$, and l_1 and h_y are both constants. From (A11) and (A12) in Appendix, we obtain the lowest order approximation equations as

$$\begin{aligned}
&(\bar{u}_0 + S_1\bar{u}_s - C_{00})\left[\frac{\mathrm{d}^2\Psi_1^{(0)}}{\mathrm{d}y^2} + F_r\left(\Psi_2^{(0)} - \bar{\psi}_1^{(0)}\right)\right] \\
&\qquad + [\beta + F_r\bar{u}_s(S_1 - S_2)]\Psi_1^{(0)} = 0, \\
&(\bar{u}_0 + S_l\bar{u}_s - C_{00})\left[\frac{\mathrm{d}^2\Psi_l^{(0)}}{\mathrm{d}y^2} + F_r\left(\Psi_{l+1}^{(0)} - 2\Psi_l^{(0)} + \Psi_{l-1}^{(0)}\right)\right] \\
&\qquad + [\beta - F_r\bar{u}_s(S_{l-1} - 2S_l + S_{l+1})]\Psi_l^{(0)} = 0, \\
&(\bar{u}_0 + S_N\bar{u}_s - C_{00})\left[\frac{\mathrm{d}^2\Psi_N^{(0)}}{\mathrm{d}y^2} - F_r\left(\Psi_N^{(0)} - \Psi_{N-1}^{(0)}\right)\right] \\
&\qquad + [\beta + F_r\bar{u}_s(S_N - S_{N-1}) + H_1 l_1 h_y]\Psi_N^{(0)} = 0, \\
&\Psi_1^{(0)} = \Psi_2^{(0)} = \cdots = \Psi_N^{(0)} = 0, \qquad y = 0,1. \\
&\qquad (l = 2,3,\cdots,N-1)
\end{aligned} \tag{10}$$

The first order equations are

$$(\bar{u}_0 + S_1\bar{u}_s - C_{00})\left[\frac{d^2\Psi_1^{(1)}}{dy^2} + F_r\left(\Psi_2^{(1)} - \Psi_1^{(1)}\right)\right] + [\beta + F_r\bar{u}_s(S_1 - S_2)]\Psi_1^{(1)}$$
$$= (C_{01} - ky)\left[\frac{d^2\Psi_1^{(0)}}{dy^2} + F_r(\Psi_2^{(0)} - \Psi_1^{(0)})\right],$$
$$(\bar{u}_0 + S_l\bar{u}_s - C_{00})\left[\frac{d^2\Psi_0^{(1)}}{dy^2} + F_r\left(\Psi_{l+1}^{(1)} - 2\Psi_l^{(1)} + \Psi_{l-1}^{(1)}\right)\right]$$
$$+ [\beta - F_r\bar{u}_s(S_{l-1} - 2S_l + S_{l+1})]\Psi_l^{(0)}$$
$$= (C_{01} - ky)\left[\frac{d^2\Psi_l^{(0)}}{dy^2} + F_r(\Psi_{l+1}^{(0)} - 2\Psi_l^{(0)} + \Psi_{l-1}^{(0)})\right], \tag{11}$$
$$(\bar{u}_0 + S_N\bar{u}_s - C_{00})\left[\frac{d^2\Psi_N^{(1)}}{dy^2} - F_r\left(\Psi_N^{(1)} - \Psi_{N-1}^{(1)}\right)\right]$$
$$+ [\beta + F_r\bar{u}_s(S_N - S_{N-1}) + H_1 l_1 h_y]\Psi_N^{(1)} = (C_{01} - ky)\left[\frac{d^2\Psi_N^{(0)}}{dy^2} - F_r(\Psi_N^{(0)} - \Psi_{N-1}^{(0)})\right],$$
$$\Psi_1^{(1)} = \Psi_2^{(1)} = \cdots = \Psi_N^{(1)} = 0, \qquad y = 0,1.$$

It is easily shown that the solutions of Eqs.(10) may be written as

$$\overline{\Psi}_l^{(0)} = A_l \sin m\pi y. \qquad (l = 1,2,\cdots,N) \tag{12}$$

Substituting (12) into (10) gives

$$\{m^2\pi^2 + F_r - [\beta + F_r\bar{u}_s(S_1 - S_2)]/(\bar{u}_0 + S_1\bar{u}_s - C_{00})\}A_1 - F_rA_2 = 0,$$
$$F_rA_{l-1} - \{m^2\pi^2 + 2F_r - [\beta - F_r\bar{u}_s(S_{l-1} - 2S_l + S_{l-1l})]/(\bar{u}_0 + S_l\bar{u}_s - C_{00})\}A_l$$
$$+ F_rA_{l+1} = 0, \tag{13}$$
$$F_rA_{N-1} - \{m^2\pi^2 + F_r - [\beta + F_r\bar{u}_s(S_N - S_{N-1}) + H_1 l_1 h_y]/(\bar{u}_0 + S_N\bar{u}_s - C_{00})\}A_N = 0.$$
$$(l = 2,3,\cdots,N-1)$$

The condition that A_l has nonzero solutions leads to an N−order algebraic equation of $\bar{u}_0 - C_{00}$ which gives N modes of Rossby wave, one of them is barotropic mode, and the others are baroclinic (Pedlosky, 1979).

For simplicity, assuming $r_l = A_l / A_1$ in Eq.(13), we obtain

$$r_1 = 1,$$
$$r_2 = (m^2\pi^2 + F_r)/F_r - [\beta + F_r\bar{u}_s(S_1 - S_2)]/[F_r(\bar{u}_0 + S_1\bar{u}_s - C_{00})],$$
$$r_{l-1} = \{(m^2\pi^2 + 2F_r)/F_r - [\beta - F_r\bar{u}_s(S_{l-1} - 2S_l + S_{l+1})]/ \tag{14}$$
$$[F_r(\bar{u}_0 + S_l\bar{u}_s - C_{00})\}r_l - r_{l+1},$$
$$r_N = r_{N-1}/\lambda,$$

where

$$\lambda = \frac{m^2\pi^2 + Fr}{F_r} - \frac{\beta + F_r\bar{u}_s(S_N - S_{N-1}) + H_1 l_1 h_y}{F_r(\bar{u}_0 + S_N\bar{u}_s - C_{00})}.$$

From the solvable condition of the first order Eq.(11), the following expression may be easily obtained

$$C_{01} = k/2.$$

From the functional forms of the right terms of (11), it is obviously seen that the special solutions of (11) may be written as

$$\overline{\Psi}_l^{(1)} = a_l(y)\sin m\pi y + b_l(y)\cos m\pi y. \tag{15}$$

Substituting them into (11) gives

$$\begin{cases} a_l(y) = \left\{Ekr_l \Big/ \left[4m^2\pi^2\left(\sum\limits_{i=1}^{N} r_i^2\right)\right]\right\} y, \\ b_l(y) = \left\{Ekr_l \Big/ \left[4m\pi \sum\limits_{i=1}^{N} r_i^2\right]\right\}(y - y^2). \end{cases} \tag{16}$$

Performing a tedious calculation of integrations (5), we obtain the coefficients of the KdV equation as follows:

$$E_1 = \frac{1}{2}\sum_{i=1}^{N} r_i^2\beta_i / u_i^2 + \frac{\delta Ek}{8m^2\pi^2\sum\limits_{i=1}^{N} r_i^2}\sum_{i=1}^{N}\frac{r_i^2\beta_i}{u_i^2},$$

$$E_2 = (4\delta k / 3m\pi)\sum_{i=1}^{N}(r_i^3\beta_i / u_i^3) + [\delta^2 Ek^2 / (2m^3\pi^3\sum_{i=1}^{N} r_i^2)]\sum_{i=1}^{N}(r_i^3\beta_i / u_i^3), \qquad m:\ \text{odd} \tag{17}$$

$$E_2 = -(5E\delta^2 k^2 / (6m^3\pi^3\sum_{i=1}^{N} r_i^2))\sum_{i=1}^{N}\frac{r_i^3\beta_i}{u_i^3} + (2\delta^2 k^2 / m\pi)\sum_{i=1}^{N}(r_i^3\beta_i / u_i^4), \qquad m:\ \text{even}$$

$$E_3 = \frac{1}{2}\sum_{i=1}^{N} r_i^2 + \delta Ek / (\delta m^2\pi^2),$$

where

$$E = \frac{\beta + F_r\bar{u}_s(S_1 - S_2)}{(\bar{u}_0 + S_1\bar{u}_s - C_{00})} + \sum_{i=2}^{N-1}\left[r_r^2\frac{\beta - F_r\bar{u}_s(S_{r-1} - 2S_r + S_{i+1})}{(\bar{u}_0 + S_1\bar{u}_s - C_{00})^2}\right] + r_N^2\frac{\beta + F_r\bar{u}_s(S_N - S_{N-1})}{(\bar{u}_0 + S_N\bar{u}_s - C_{00})^2},$$

$$u_i = \bar{u}_0 + S_i\bar{u}_s - C_{00}, \qquad \beta_1 = \beta + F_r(S_1 - S_2)\bar{u}_s,$$

$$\beta_l = \beta - F_r(S_{l-1} - 2S_l + S_{l+1})\bar{u}_s, \qquad \beta_N = \beta - F_r(S_{N-1} - S_N)\bar{u}_s + H_1 l_1 h_y.$$

$$(l = 2,3,\cdots N-1)$$

Hukuda has shown in his study of the two–level model that if the basic current has no vertical shear, the barotropic and baroclinic modes would become the pure barotropic ($r_1 = 1$) and baroclinic modes ($r_2 = -1$) respectively, and in this case the KdV equation could not describe the pure baroclinic solitary waves on the flow without vertical shear since $E_2 = 0$, and it is necessary to introduce the so–called MKdV equation. However, from the expression of E_2 in (17) it is easily known that for the multilevel model including topography the case of $E_2 = 0$ cannot exist even if the flow has no vertical shear. In other words, the KdV equation in this case may still describe the barotropic and baroclinic solitary waves, and the MKdV equation need not be introduced. We will give the calculating results for the above case.

Now, the streamline patterns of solitary waves at each level may be calculated by using the following expression:

$$\psi_l(\xi,y) = -\int_0^y \bar{u}_l(y)\mathrm{d}y + \varepsilon\,\mathrm{sgn}(E_2E_3)\mathrm{sech}^2\left(\left|\frac{E_2}{12E_3}\right|^{1/2}\xi\right)(\Psi_l^{(0)} + \delta\Psi_l^{(1)}), \quad (l = 1,2,\cdots,N) \tag{18}$$

IV. EXAMPLES

We discuss the effects of topography on the solitary Rossby waves in terms of a simple three–level model. Assume the basic flow in the form

$$\bar{u}_1 = \bar{u}_0 + S_1\bar{u}_s + \delta ky,$$
$$\bar{u}_2 = \bar{u}_0 + S_2\bar{u}_2 + \delta ky,$$
$$\bar{u}_3 = \bar{u}_0 + \delta ky.$$

Two types of vertical shears of flow, namely, D–shear ($S_1 = 0$, $S_2 = 1$) and non–shear ($S_1 = 0$, $S_2 = 0$) are taken with emphasis on topographical effects. The horizontal shear of flow is taken to be cyclonic ($k = -1$) and anticyclonic ($k = 1$). For the topography we calculate separately the southern–slope ($l_1 = 1$) and northern–slope ($l_1 = -1$). In addition, we consider the effects of Froude number, and calculate two cases of $F_r = 0.5$ and $F_r = 0.2$. In the all calculations, we take $\bar{u}_0$, $\bar{u}_s = 1$, $\varepsilon = 0.1$, $\delta = 0.001$ and $h_y = 1$.

1. *Effects of Horizontal Shear of Flow*

Hukuda did not discuss the influence of the horizontal shear of the basic flow in his work. In the paper (Lu, 1987), we only discussed the cases of the basic flow with anticyclonic shear and the effects of the easterly and westerly flows, and in another paper (Lu, 1988), we emphatically studied the effects of the flow with weak quadric meridional shear. The results of calculations show that the variation of horizontal shear pattern of flow has no effect on the solitary wave with even meridional wavenumber m, but it has important influence on the solitary wave with odd m, and may cause violent change of solitary wave pattern, becoming a completely opposite form. The change of streamline pattern is independent of the topography if topographic height is about 1 km. As shown in the paper (Lu, 1987), if the characteristic height of topography is quite high (about 4km) the solitary wave patterns may also change for the flow with anticyclonic shear (see Fig. 1 in Lu (1987)). In addition, for the easterly and westerly flows with the same horizontal shear form the solitary wave patterns may also change (see Fig. 2 in Lu (1987)). For the present, only the influences of westerly basic flow with different shear forms and topography with height about 1 km will be considered. Comparing Fig. 1a and Fig. 3 in Lu (1987) to Fig. 1a and Fig.2a in this paper respectively, we easily see that they are similar , and that the barotropic model may show some primary features. The calculations also show that the effect of horizontal shear of flow on the solitary waves with odd m is irrelevant to the solitary wave modes. When the horizontal shear form changes from anticyclonic ($k = 1$) to cyclonic ($k = -1$), the streamline patterns of solitary wave with $m = 1$ will transform from ridge to trough at all levels (Figs. 1a, 1b). The changes are similar for all the different solitary wave modes. Figs. 1c and 1d give the streamline patterns of $m = 3$ for the different horizontal shears. When the shear pattern changes from anticyclonic to cyclonic, the streamline patterns are completely opposite. However, the change of horizontal shear form has no influence on the solitary wave pattern for the case

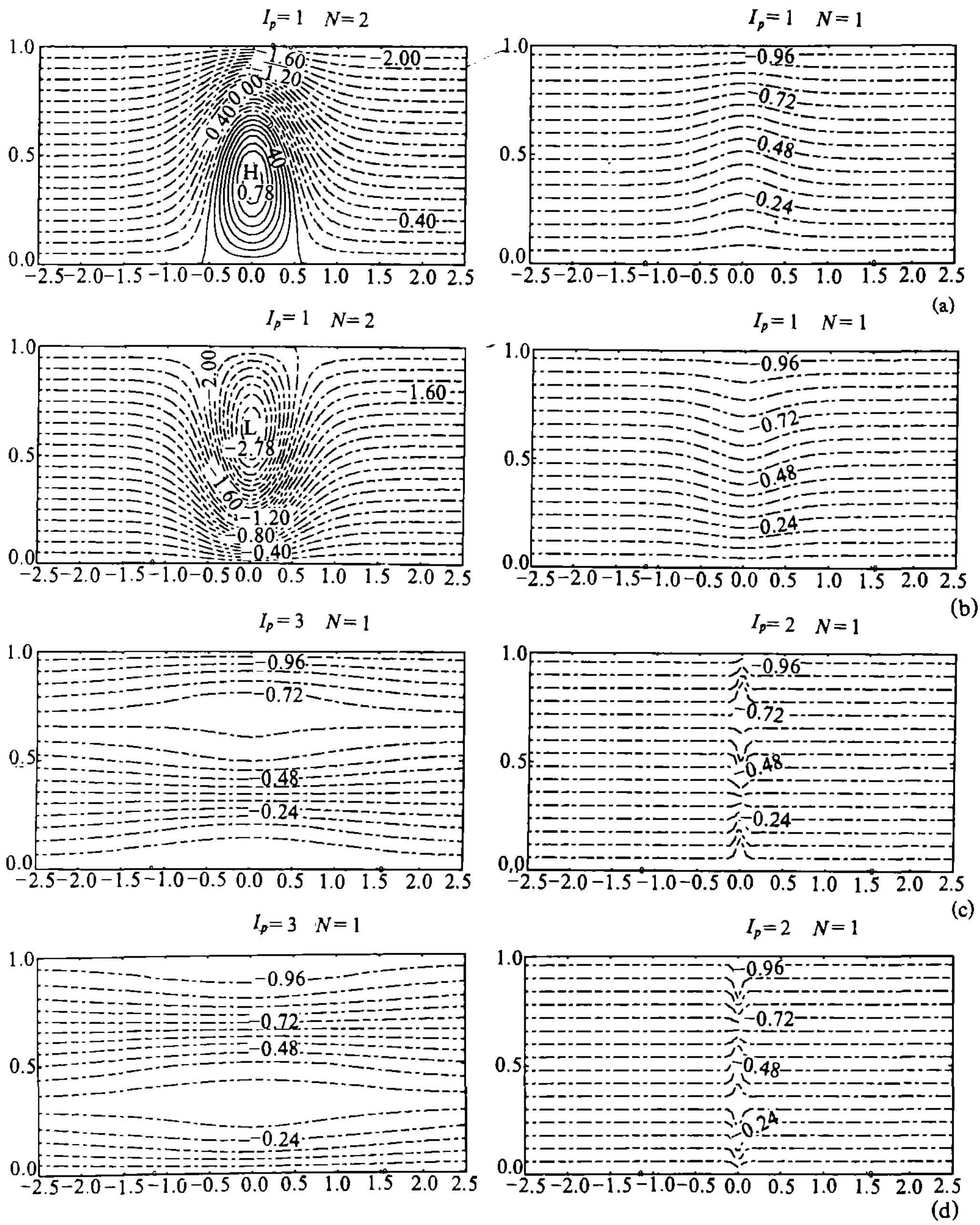
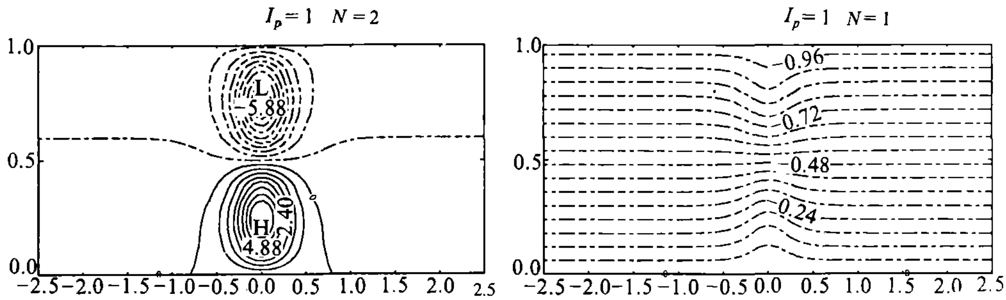
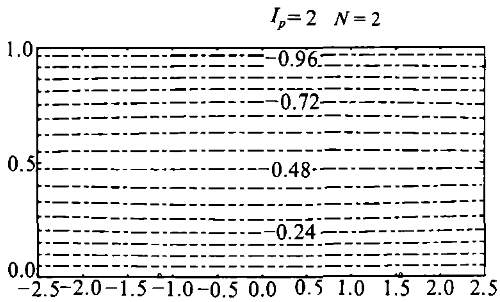


Fig. 1. The streamline patterns of solitary Rossby wave for meridional wavenumber $m=1$ and 3. ($F_r=0.5$, D–shear pattern). (a) Anticyclonic horizontal shear and southern slope of topography; (b) Cyclonic shear and southern slope; (c) Anticyclonic shear and northern slope; (d) Cyclonic shear and northern slope. (a) and (b) : $m=1$, barotropic modes; (c) and (d) : $m=3$, upper figures —baroclinic modes 1, lower figures — baroclinic modes 2.

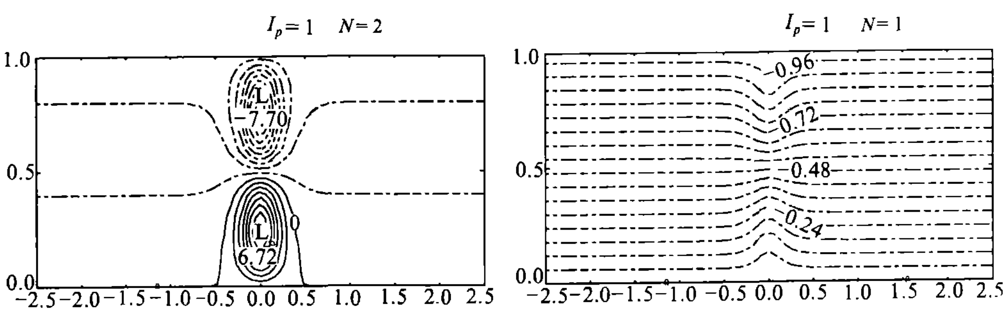


(a)

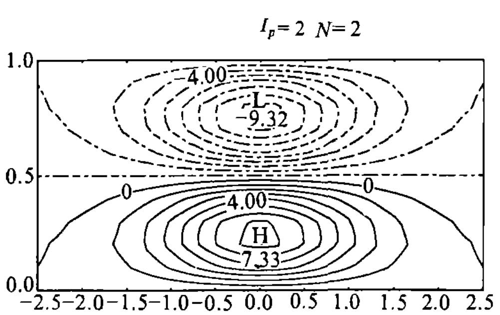


(b)

Fig. 2. The streamline patterns of solitary Rossby wave for meridional wave number $m=2$. (Anticyclonic shear, southern slope of topography, basic flow with no vertical shear). (a) barotropic mode; (b) baroclinic mode 1.



(a)



(b)

Fig. 3. As in Fig. 2 but for flow with D−vertical shear.

of even m. It is easily known from the expression of coefficients of the KdV equation (17) that the forms of solitary waves are mainly determined by the nonlinear terms E_2 . For the case of odd m, the first term in the expression of E_2 plays a decisive role since δ is a very small value, therefore, the sign of k determines that of E_2, which, in turn, determines that of the streamline pattern of solitary waves. And on the contrary, for the solitary wave with even m, the change in sign of k has no effect on E_2, and thus there is no influence on the solitary wave pattern.

2. *Effects of Vertical Shear of the Basic Flow*

Hukuda (1979) has shown that the vertical shear of flow has a tendency to steepen the solitary waves. Our calculations show that the influences of vertical shear are complicated, it generally steepens solitary waves(especially for barotropic modes), but its effect also depends on the specific mode, the level of geopotential surface, the value of F_r, and the slope of topography.

Fig. 2 shows the patterns of solitary waves for $m=2$ in the case of basic flow with no vertical shear and topography with southern slope. Fig. 3 gives streamline patterns in the case of flow with vertical shear. Comparison of Fig. 2 with Fig. 3a shows that the vertical shear of flow steepens obviously the steepness of barotropic solitary modes, and for the baroclinic mode 1 of solitary waves at mid–level it may cause the streamline patterns to change greatly (see Fig. 2 and Fig. 3b), and for the baroclinic mode 2, on the contrary, the steepness of solitary waves decreases to some extent (figures not shown). The results of calculations also show that for the solitary waves with $m=1$ the vertical shear increases the steepness of solitary wave in the area of southern slope of topography, and decreases that in the area of topographic northern slope. For the case of $m=2$, the results are more complicated in the area of northern slope than that of southern slope.

3. *Influences of Froude Number*

Hukuda (1979) has also shown in his paper that the steepness of solitary wave increases with decreasing Froude number F_r. However, our calculations show that the influence of F_r on the steepness of solitary wave is complicated, it depends on the meridional wavenumber, and the specific mode. For $m=1$, as shown by Hukuda, the steepness of solitary wave increases with decreasing F_r (Fig.4), the conclusions are also the same for the barotropic mode and baroclinic mode 1 in the case of $m=2$. However, for the baroclinic mode 2, the steepness of upper solitary waves decreases with decreasing F_r(figure not shown). For the case of $m=3$, the situation is more complicated.

4. *Effects of Southern and Northern Slopes of Topography*

The solitary wave patterns are generally different in the area of southern and northern slope of topography. The parameters in Figs. 5 and 2 are the same but topographic slope. Fig.2 is the results for the southern slope , and Fig. 5 for the northern slope. Comparing these two figures shows that the patterns of solitary waves are quite different. For the northern slope, the streamlines of barotropic mode are straight (figure not shown), and for southern slope the streamline patterns are those of the lower in the north and higher in the south. For the baroclinic mode 1, the solitary wave patterns in the northern slope are those of the higher in north, and the lower in south, and in the southern slope they are straight. For the baroclinic mode 2, the streamlines are straight in the southern slope, and in the area of northern slope they are those of the lower in south and the higher in north. Generally, the influences of northern slope on the solitary wave patterns are greater than that of southern slope, and the value of topographic slope also has influence, however, its effects are small compared to that of the form of topographic slope.

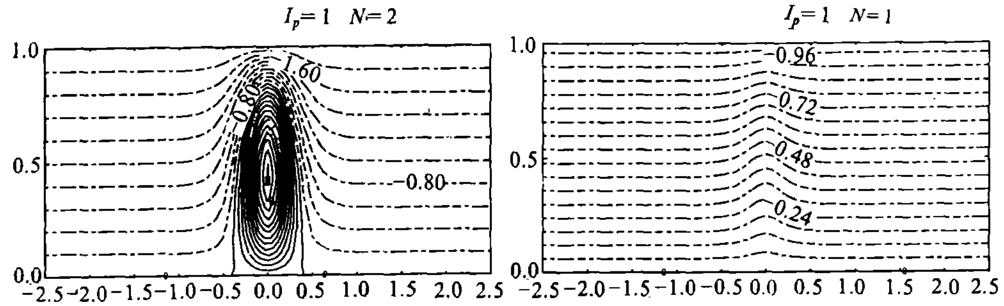


Fig. 4. As in Fig. 1a, but for $F_1 = 0.2$

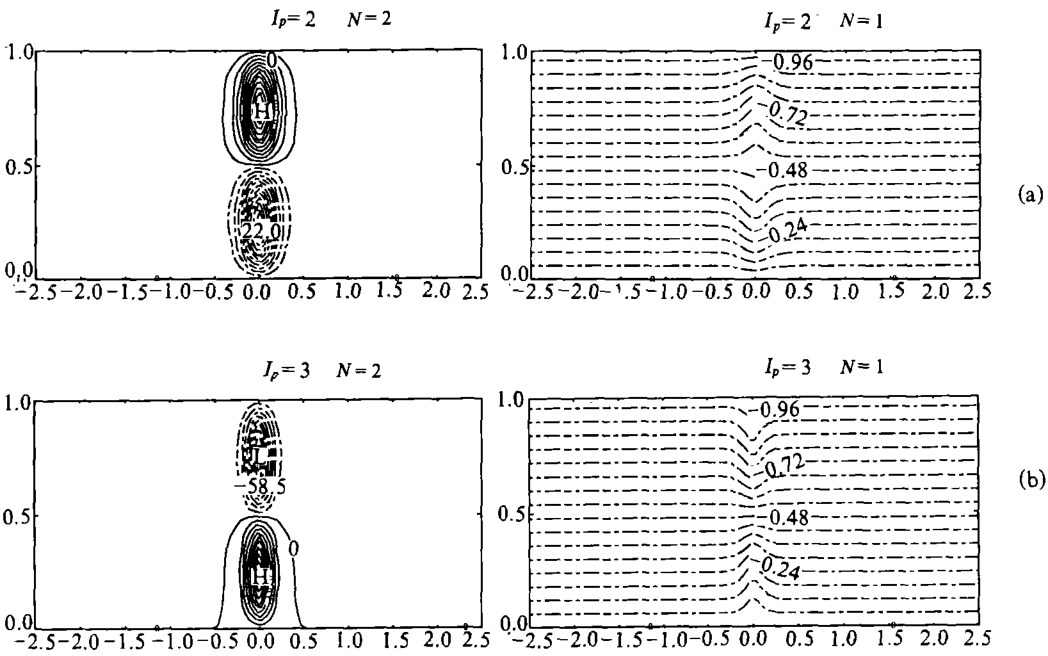


Fig. 5. As in Fig. 2a, but for northern slope of topography: (a) baroclinic mode 1;(b) baroclinic mode 2.

In addition, the different modes of solitary wave generally have different steepnesses, and in some cases there even occurs a completely different streamline pattern (in Fig. 5). The solitary waves at different levels usually have different forms or steepnesses.

V. CONCLUSIONS

(1) The KdV equation including topography in an N-level model may describe solitary Rossby waves in the case of basic flow with no vertical shear, and it is not necessary to introduce the MKdV equation.

(2) The change of horizontal shear form of basic flow has no influence on the streamline pattern of solitary Rossby wave if its meridional wavenumber m is even. But for the solitary

wave with odd m, it has tendency to transform the solitary wave pattern into a completely opposite form, and the transforming is independent of topography (if the topography height is lower).

(3) The effects of vertical shear of flow on the solitary wave depend not only on the specific mode and Froude number but also on the meridional wavenumber and topography. For the barotropic mode, the vertical shear has the tendency of steepening the solitary wave, and for the baroclinic modes, the vertical shear may increase the steepness of solitary wave or decrease the steepness and even change completely the characters of solitary wave, all of those depend on the meridional wavenumber, specific mode and southern or northern slope of topography.

(4) The influence of Froude number on the steepness of solitary wave is important. For the barotropic mode and baroclinic mode 1 with $m=1$ and $m=2$, the steepness of solitary wave increases with decreasing Froude number F_r. However, for the baroclinic mode 2 with $m=2$, the steepness of solitary wave at upper level decreases with decreasing F_r. For $m=3$, the situations are more complicated. All the results show that effects of Froude number are complex, and are not as simple as shown in Hukuda's work.

(5) The effects of topography on the solitary waves are also important, its influences are mainly caused by the southern or northern slope forms of topography, the value of topographic slope has influence but it is not as large as that of slope form. The northern slope is a more important factor, and only in the area of northern slope the solitary dipole pattern similar to the blocking situation may be caused.

(6) The solitary waves for different modes and levels generally have different steepnesses. The barotropic mode is usually the most obvious, the baroclinic mode 1 is the second, and the mode 2 is the weakest. The results of calculations also show that the solitary wave located at mid-level is generally the strongest, and the one located at upper level is the weakest.

APPENDIX

Utilizing the transform (3), from (1) we obtain

$$\frac{\partial}{\partial x'}(\psi_1' + Cy')\frac{\partial}{\partial y'}[\nabla^2\psi_1' + \beta' y' + F_r(\psi_2' - \psi_1')] - \frac{\partial}{\partial y'}(\psi_1' + Cy')\frac{\partial}{\partial x'}[\nabla^2\psi_1' + \beta' y' + F_r(\psi_2' - \psi_1')] = 0,$$

$$\begin{aligned} &\frac{\partial}{\partial x'}(\psi_l' + Cy')\frac{\partial}{\partial y'}[\nabla^2\psi_l' + \beta' y' + F_r(\psi_{l+1}' - 2\psi_l' + \psi_{l-1}')] \\ &\qquad - \frac{\partial}{\partial y'}(\psi_l' + Cy')\frac{\partial}{\partial x'}[\nabla^2\psi_l' + \beta' y' + F_r(\psi_{l+1}' - 2\psi_l' + \psi_{l-1}')] = 0, \end{aligned} \tag{A1}$$

$$\begin{aligned} &\frac{\partial}{\partial x'}(\psi_N' + Cy')\frac{\partial}{\partial y'}[\nabla^2\psi_N' + \beta' y' - F_r(\psi_N' - \psi_{N-1}') + H_1 h'] \\ &\qquad - \frac{\partial}{\partial y'}(\psi_N' + Cy')\frac{\partial}{\partial x'}[\nabla^2\psi_N' + \beta' y' - F_r(\psi_N' - \psi_{N-1}') + H_1 h'] = 0, \end{aligned}$$

$$(l = 2,3,\cdots,N-1)$$

In order to obtain KdV equation, we make the following transformation:

$$\xi=\varepsilon^{1/2}x', \qquad y=y', \tag{A2}$$

where $\varepsilon(\ll 1)$ is a small parameter of amplitude.

From (A1) we obtain (omitting primes "$'$ ")

$$\begin{aligned}
&\left[\frac{\partial\psi_1}{\partial\xi}\frac{\partial}{\partial y}-\left(\frac{\partial\psi_1}{\partial y}+C\right)\frac{\partial}{\partial\xi}\right]\left[\varepsilon\frac{\partial^2\psi_1}{\partial\xi^2}+\frac{\partial^2\psi_1}{\partial y^2}+\beta y+F_r(\psi_2-\psi_1)\right]=0,\\
&\left[\frac{\partial\psi_l}{\partial\xi}\frac{\partial}{\partial y}-\left(\frac{\partial\psi_l}{\partial y}+C\right)\frac{\partial}{\partial\xi}\right]\left[\varepsilon\frac{\partial^2\psi_l}{\partial\xi^2}+\frac{\partial^2\psi_l}{\partial y^2}+\beta y+F_r(\psi_{l+1}-2\psi_l+\psi_{l-1})\right]=0,\\
&\left[\frac{\partial\psi_N}{\partial\xi}\frac{\partial}{\partial y}-\left(\frac{\partial\psi_N}{\partial y}+C\right)\frac{\partial}{\partial\xi}\right]\left[\varepsilon\frac{\partial^2\psi_N}{\partial\xi^2}+\frac{\partial^2\psi_N}{\partial y^2}+\beta y-F_r(\psi_N-\psi_{N-1})+H_1h\right]=0,\\
&\qquad (l=2,3,\cdots,N-1),
\end{aligned}\tag{A3}$$

and

$$\frac{\partial\psi_l}{\partial\xi}=0, \qquad y=0,1. \tag{A4}$$

Expand the variable ψ and the parameter C in power of ε as follows

$$\begin{aligned}
&\psi_l(\xi,y)=-\int_0^y\bar{u}_l(y)\mathrm{d}y+\varepsilon\psi_l^{(1)}(\xi,y)+\varepsilon^2\psi_l^{(2)}(\xi,y)+\cdots,\\
&C=C_0+\varepsilon C_1+\varepsilon^2C_2+\cdots,
\end{aligned}\tag{A5}$$

where $\bar{u}_l\ (y)$ is basic flow at lth–level. Substituting (A5) into (A3) leads to following approximation equations.

The ε'–order approximation equations are

$$\begin{aligned}
&\frac{\partial\psi_1^{(1)}}{\partial\xi}\left[-\frac{\partial^2\bar{u}_1}{\partial y^2}+\beta+F_r(\bar{u}_1-\bar{u}_2)\right]+(\bar{u}_1-C_0)\left[\frac{\partial^3\psi_1^{(1)}}{\partial\xi\partial y^2}+F_r\left(\frac{\partial\psi_2^{(1)}}{\partial\xi}-\frac{\partial\psi_1^{(1)}}{\partial\xi}\right)\right]=0,\\
&\frac{\partial\psi_l^{(1)}}{\partial\xi}\left[-\frac{\partial^2\bar{u}_l}{\partial y^2}+\beta+F_r(-\bar{u}_{l+1}+2\bar{u}_l-\bar{u}_{l-1})\right]\\
&\qquad+(\bar{u}_l-C_0)\left[\frac{\partial^3\psi_l^{(1)}}{\partial\xi\partial y^2}+F_r\left(\frac{\partial\psi_{l+1}^{(1)}}{\partial\xi}-2\frac{\partial\psi_l^{(1)}}{\partial\xi}+\frac{\partial\psi_{l-1}^{(1)}}{\partial\xi}\right)\right]=0,\\
&\frac{\partial\psi_N^{(1)}}{\partial\xi}\left[-\frac{\partial^2\bar{u}_N}{\partial y^2}+\beta-F_r(\bar{u}_{N-1}-\bar{u}_N)+H_1\frac{\mathrm{d}h}{\mathrm{d}y}\right]\\
&\qquad+(\bar{u}_N-C_0)\left[\frac{\partial^3\psi_N^{(1)}}{\partial\xi\partial y^2}-F_r\left(\frac{\partial\psi_N^{(1)}}{\partial\xi}-\frac{\partial\psi_{N-1}^{(1)}}{\partial\xi}\right)\right]=0,
\end{aligned}\tag{A6}$$

and

$$\frac{\partial\psi_l^{(1)}}{\partial\xi}=0, \qquad y=0,1 \tag{A7}$$

the ε^2–order equations are .

$$\frac{\partial\psi_1^{(2)}}{\partial\xi}\left[-\frac{\partial^2\bar{u}_1}{\partial y^2}+\beta+F_r(\bar{u}_1-\bar{u}_2)\right]+(\bar{u}_1-C_0)\left[\frac{\partial^3\psi_1^{(2)}}{\partial\xi\partial y^2}+F_r\left(\frac{\partial\psi_2^{(2)}}{\partial\xi}-\frac{\partial\psi_1^{(2)}}{\partial\xi}\right)\right]$$

$$=-(\bar{u}_1-C_0)\frac{\partial^3\psi_1^{(1)}}{\partial\xi^3}-\frac{\partial\psi_1^{(1)}}{\partial\xi}\left[\frac{\partial^3\psi_1^{(1)}}{\partial y^3}+F_r\left(\frac{\partial\psi_2^{(1)}}{\partial y}-\frac{\partial\psi_1^{(1)}}{\partial y}\right)\right]$$

$$+\left(\frac{\partial\psi_1^{(1)}}{\partial y}+C_1\right)\left[\frac{\partial^3\psi_1^{(1)}}{\partial\xi\partial y^2}+F_r\left(\frac{\partial\psi_2^{(1)}}{\partial\xi}-\frac{\partial\psi_1^{(1)}}{\partial\xi}\right)\right],$$

$$\frac{\partial\psi_l^{(2)}}{\partial\xi}\left[-\frac{\partial^2\bar{u}_l}{\partial y^2}+\beta+F_r(-\bar{u}_{l+1}+2\bar{u}_l-\bar{u}_{l-1})\right]+(\bar{u}_l-C_0)\left[\frac{\partial^3\psi_l^{(2)}}{\partial\xi\partial y^2}+F_r\left(\frac{\partial\psi_{l+1}^{(1)}}{\partial\xi}-2\frac{\partial\psi_l^{(2)}}{\partial\xi}+\frac{\partial\psi_{l-1}^{(2)}}{\partial\xi}\right)\right]$$

$$=-(\bar{u}_l-C_0)\frac{\partial^3\psi_l^{(l)}}{\partial\xi^3}-\frac{\partial\psi_l^{(1)}}{\partial\xi}\left[\frac{\partial^3\psi_l^{(1)}}{\partial y^3}+F_r\left(\frac{\partial\psi_{l+1}^{(1)}}{\partial y}-2\frac{\partial\psi_l^{(1)}}{\partial y}+\frac{\partial\psi_{l-1}^{(1)}}{\partial y}\right)\right] \tag{A8}$$

$$+\left(\frac{\partial\psi_l^{(1)}}{\partial y}+C_l\right)\left[\frac{\partial^3\psi_l^{(1)}}{\partial\xi\partial y^2}+F_r\left(\frac{\partial\psi_{l+1}^{(1)}}{\partial\xi}-2\frac{\partial\psi_l^{(1)}}{\partial\xi}+\frac{\partial\psi_{l-1}^{(1)}}{\partial\xi}\right)\right],$$

$$\frac{\partial\psi_N^{(2)}}{\partial\xi}\left[-\frac{\partial^2\bar{u}_N}{\partial y^2}+\beta-F_r(\bar{u}_{N-1}-\bar{u}_N)+H_1\frac{dh}{dy}\right]+(\bar{u}_N-C_0)\left[\frac{\partial^3\psi_N^{(2)}}{\partial\xi\partial y^2}-F_r\left(\frac{\partial\psi_N^{(2)}}{\partial\xi}-\frac{\partial\psi_{N-1}^{(2)}}{\partial\xi}\right)\right]$$

$$=-(\bar{u}_N-C_0)\frac{\partial^3\psi_N^{(1)}}{\partial\xi^3}-\frac{\partial\psi_N^{(1)}}{\partial\xi}\left[\frac{\partial^3\psi_N^{(1)}}{\partial y^3}-F_r\left(\frac{\partial\psi_N^{(1)}}{\partial y}-\frac{\partial\psi_{N-1}^{(1)}}{\partial y}\right)\right]$$

$$+\left(\frac{\partial\psi_N^{(1)}}{\partial y}+C_1\right)\left[\frac{\partial^3\psi_N^{(1)}}{\partial\xi\partial y^2}-F_r\left(\frac{\partial\psi_N^{(1)}}{\partial\xi}-\frac{\partial\psi_{N-1}^{(1)}}{\partial\xi}\right)\right],$$

$$(l=2,3,\cdots,N-1)$$

and

$$\frac{\partial\psi_l^{(2)}}{\partial\xi}=0,\qquad y=0,1. \tag{A9}$$

It is easy to know that (A6) and (A7) are separable with respect to ξ and y, and for this purpose, setting

$$\psi_l^{(1)}=\Phi(\xi)\Psi_l(y),\qquad (l=1,2,\cdots,N) \tag{A10}$$

and substituting it into (A6) and (A7) gives the eigenvalue problem of Ψ_l:

$$(\bar{u}_1-C_0)\left[\frac{d^2\Psi_1}{dy^2}+F_r(\Psi_2-\Psi_1)\right]+\left[\beta-\frac{d^2\bar{u}_1}{dy^2}+F_r(\bar{u}_1-\bar{u}_2)\right]\Psi_1=0,$$

$$(\bar{u}_l-C_0)\left[\frac{d^2\Psi_l}{dy^2}+F_r(\Psi_{l+1}-2\Psi_l+\Psi_{l-1})\right]+\left[\beta-\frac{d^2\bar{u}_l}{dy^2}-F_r(\bar{u}_{l-1}-2\bar{u}_l+\bar{u}_{l+1})\right]\Psi_l=0, \tag{A11}$$

$$(\bar{u}_N-C_0)\left[\frac{d^2\Psi_N}{dy^2}-F_r(\Psi_N-\Psi_{N-1})\right]+\left[\beta-\frac{d^2\bar{u}_N}{dy^2}-F_r(\bar{u}_{N-1}-\bar{u}_N)+H_1\frac{dh}{dy}\right]\Psi_N=0,$$

$$\Psi_l(y)=0,\qquad y=0,1. \tag{A12}$$

Equations (A11) describe the meridional structure of Rossby waves including topographic effect. To determine $\Phi(\xi)$, utilizing (A8), then substituting (A10) into the right hand of (A8), and omitting the cases of $u_l=C_0$, from (A11) we obtain

$$
\begin{aligned}
&\frac{\partial\psi_1^{(2)}}{\partial\xi}\left[\beta-\frac{d^2\bar{u}_1}{dy^2}+F_r(\bar{u}_1-\bar{u}_2)\right]+(\bar{u}_1-\bar{C}_0)\left[\frac{\partial^3\psi_1^{(2)}}{\partial\xi\partial y^2}+F_r\left(\frac{\partial\psi_2^{(2)}}{\partial\xi}-\frac{\partial\psi_1^{(2)}}{\partial\xi}\right)\right] \\
&\quad=-(\bar{u}_1-C_0)\Psi_1\frac{d^3\Phi}{d\xi^3}+\Psi_1^2\Phi\frac{d\Phi}{d\xi}\frac{d}{dy}\left[\frac{\beta-\frac{d^2\bar{u}_1}{dy^2}+F_r(\bar{u}_1-\bar{u}_2)}{\bar{u}_1-C_0}\right] \\
&\quad-C_1\Psi_1\frac{d\Phi}{d\xi}\left[\frac{\beta-\frac{d^2\bar{u}_1}{dy^2}+F_r(\bar{u}_1-\bar{u}_2)}{\bar{u}_1-C_0}\right], \\
&\frac{\partial\psi_l^{(2)}}{\partial\xi}\left[\beta-\frac{d^2\bar{u}_l}{dy^2}-F_r(\bar{u}_{l+1}-2\bar{u}_l+\bar{u}_{l-1})\right]+(\bar{u}_l-C_0)\left[\frac{\partial^3\psi_l^{(2)}}{\partial\xi\partial y^2}+F_r\left(\frac{\partial\psi_{l+1}^{(2)}}{\partial\xi}-2\frac{\partial\psi_l^{(2)}}{\partial\xi}+\frac{\partial\psi_{l-1}^{(2)}}{\partial\xi}\right)\right] \\
&\quad=-(\bar{u}_l-C_0)\Psi_l\frac{d^3\Phi}{d\xi^3}+\Psi_l^2\Phi\frac{d\Phi}{d\xi}\frac{d}{dy}\left[\frac{\beta-\frac{d^2\bar{u}_l}{dy^2}-F_r(\bar{u}_{l-1}-2\bar{u}_l+\bar{u}_{l+1})}{\bar{u}_l-C_0}\right] \qquad \text{(A13)} \\
&\quad-C_1\Psi_l\frac{d\Phi}{d\xi}\left[\frac{\beta-\frac{d^2\bar{u}_l}{dy^2}-F_r(\bar{u}_{l-1}-2\bar{u}_l+\bar{u}_{l+1})}{\bar{u}_l-C_0}\right], \\
&\frac{\partial\psi_N^{(2)}}{\partial\xi}\left[\beta-\frac{d^2\bar{u}_N}{dy^2}-F_r(\bar{u}_{N-1}-\bar{u}_N)+H_1\frac{dh}{dy}\right]+(\bar{u}_N-C_0)\left[\frac{\partial^3\psi_N^{(2)}}{\partial\xi\partial y^2}-F_r\left(\frac{\partial\psi_N^{(2)}}{\partial\xi}-\frac{\partial\psi_{N-1}^{(2)}}{\partial\xi}\right)\right] \\
&\quad=-(\bar{u}_N-C_0)\Psi_N\frac{d^3\Phi}{d\xi^3}+\Psi_N\Phi\frac{d\Phi}{d\xi}\frac{d}{dy}\left[\frac{\beta-\frac{d^2\bar{u}_N}{dy^2}-F_r(\bar{u}_{N-1}-\bar{u}_N)+H_1\frac{dh}{dy}}{\bar{u}_N-C_0}\right] \\
&\quad-C_1\Psi_N\frac{d\Phi}{d\xi}\left[\frac{\beta-\frac{d^2\bar{u}_N}{dy^2}-F_r(\bar{u}_{N-1}-\bar{u}_N)+H_1\frac{dh}{dy}}{\bar{u}_N-C_0}\right],
\end{aligned}
$$

The condition for the existence of solutions of (A13) is that the amplitude $\Phi(\xi)$ satisfies the KdV equation (4).

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