PHYSICAL PROPERTIES OF THE SYMMETRIC AND ANTISYMMETRIC MOTIONS AND RELATED ENERGY CONVERSION

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ABSTRACT

The physical properties of the symmetric and antisymmetric motions, such as the conservation of the absolute angular momentum, the mutual conversions between various forms of energy, have been analysed by using the sets of equations in p-coordinates controlling the motions in the primitive equation model atmosphere. The results show that only the symmetric component of zonal geopotential difference caused by orography and that of the zonal frictional torque, have contribution to the change of the global angular momentum, and that the mutual conversions between various forms of energy, in addition to those similar to the results in the classical case, include those associated with the symmetric and antisymmetric motions.

Key words: symmetric and antisymmetric motions, physical properties, symmetric and antisymmetric energy, energy conversion

I. INTRODUCTION

By analysing climatic data it was found in articles(Liao,1985; 1986) that a lot of meteorological elements were mainly symmetrically distributed with respect to the equator, such as the geopotentials, temperatures, zonal mean west winds at 500 and 1000 hPa levels. Considering that global long-term weather anomalies are closely related to global long-term abnormal antisymmetric distribution of relevant meteorological elements, it seems that to study the latter would definitely contribute to understanding the former. For this purpose Liao and Zou (1987) derived the equations controlling symmetric motion and those controlling antisymmetric motion in the barotropic filtered model atmosphere, and the corresponding energy budget equations and energy conversions between the two kinds of motions. In addition, they pointed out that the asymmetric distribution of orography and that of horizontal diffusion coefficient may be the cause for producing asymmetric motion. Furthermore, the author (1985) discussed the symmetric and antisymmetric motions in the primitive equation model, presented the dynamic and thermodynamic equations describing the motions and analysed their physical properties.

This paper is a continuation of those articles previously mentioned. It aims to discuss the mutual conversions between various forms of energy associated with the two kinds of motions.

II. EQUATION SETS CONTROLLING SYMMETRIC AND ANTISYMMETRIC MOTIONS

According to Liao (1985), we have the following equation sets controlling the symmetric and antisymmetric motions in the primitive equation model atmosphere.

(1) *Symmetric Motion*

$$\frac{\partial \boldsymbol{V}_S}{\partial t}+(\boldsymbol{V}_S\cdot\nabla)\boldsymbol{V}_S+(\boldsymbol{V}_A\cdot\nabla)\boldsymbol{V}_A+\omega_S\frac{\partial \boldsymbol{V}_S}{\partial p}+\omega_A\frac{\partial \boldsymbol{V}_A}{\partial p}$$

$$+2\boldsymbol{\Omega}\times\boldsymbol{V}_S=-\nabla\phi_S+\boldsymbol{F}_S \quad , \tag{1}$$

$$\nabla\cdot\boldsymbol{V}_S+\frac{\partial\omega_S}{\partial p}=0 \quad , \tag{2}$$

$$\frac{\partial T_S}{\partial t}+\boldsymbol{V}_S\cdot\nabla\boldsymbol{T}_S+\boldsymbol{V}_A\cdot\nabla\boldsymbol{T}_A+\omega_S\frac{\partial T_S}{\partial p}+\omega_A\frac{\partial T_A}{\partial p}$$

$$-\frac{R}{c_p p}(T_S\omega_S+T_A\omega_A)=\frac{q_S}{c_p} \quad , \tag{3}$$

$$\frac{\partial\phi_S}{\partial p}=-\frac{R}{p}T_S \quad . \tag{4}$$

(2) *Antisymmetric Motion*

$$\frac{\partial \boldsymbol{V}_A}{\partial t}+(\boldsymbol{V}_S\cdot\nabla)\boldsymbol{V}_A+(\boldsymbol{V}_A\cdot\nabla)\boldsymbol{V}_S+\omega_S\frac{\partial \boldsymbol{V}_A}{\partial p}+\omega_A\frac{\partial \boldsymbol{V}_S}{\partial p}$$

$$+2\boldsymbol{\Omega}\times\boldsymbol{V}_A=-\nabla\phi_A+\boldsymbol{F}_A \quad , \tag{5}$$

$$\nabla\cdot\boldsymbol{V}_A+\frac{\partial\omega_A}{\partial p}=0 \quad , \tag{6}$$

$$\frac{\partial T_A}{\partial t}+\boldsymbol{V}_S\cdot\nabla T_A+\boldsymbol{V}_A\cdot\nabla T_S+\omega_S\frac{\partial T_A}{\partial p}+\omega_A\frac{\partial T_S}{\partial p}$$

$$-\frac{R}{c_p p}(T_S\omega_A+T_A\omega_S)=\frac{q_A}{c_p} \quad , \tag{7}$$

$$\frac{\partial\phi_A}{\partial p}=-\frac{R}{p}T_A \quad . \tag{8}$$

Here any scalar quantity M can be expressed by

$$M(\lambda,\psi,p,t)=M_S(\lambda,\psi,p,t)+M_A(\lambda,\psi,p,t), \tag{9}$$

where subscripts S and A stand for the symmetric and antisymmetric components of M; M_S and M_A have the following relations respectively:

$$M_S(\lambda,\varphi,p,t)=M_S(\lambda,-\varphi,p,t) \quad , \tag{10}$$

$$M_A(\lambda,\varphi,p,t)=-M_A(\lambda,-\varphi,p,t) \quad . \tag{11}$$

As for wind $\boldsymbol{V}$ and frictional force $\boldsymbol{F}$, we have

$$\boldsymbol{V}(\lambda,\varphi,p,t)=\boldsymbol{V}_S(\lambda,\varphi,p,t)+\boldsymbol{V}_A(\lambda,\varphi,p,t) \tag{12}$$

and

$$\boldsymbol{F}(\lambda,\varphi,p,t)=\boldsymbol{F}_S(\lambda,\varphi,p,t)+\boldsymbol{F}_A(\lambda,\varphi,p,t) \quad , \tag{13}$$

where $\boldsymbol{V}_S=u_S\boldsymbol{i}+v_A\boldsymbol{j}, \boldsymbol{V}_A=u_A\boldsymbol{i}+v_S\boldsymbol{j}; \boldsymbol{F}_S=(F_\lambda)_S\boldsymbol{i}+(F_\varphi)_A\boldsymbol{j}, \boldsymbol{F}_A=(F_\lambda)_A\boldsymbol{i}+(F_\varphi)_S\boldsymbol{j}$. In the above, the symbols which have not been explained are conventional in meteorology.

It can be seen from Eqs. (1)—(8) that symmetric and antisymmetric motions are mutually interacted and restricted.

III. TIME VARIATION IN ANGULAR MOMENTUM

As is known, the angular momentum per unit mass m can be expressed by

$$m = a(a\Omega\cos\varphi + u)\cos\varphi \quad , \tag{14}$$

and

$$\frac{dm}{dt} = -\frac{\partial\phi}{\partial\lambda} + a\cos\varphi\cdot F_\lambda \quad . \tag{15}$$

Using the continuity equation, the above equation can be written in the form

$$\frac{\partial m}{\partial t} + \nabla\cdot m\boldsymbol{V} + \frac{\partial m\omega}{\partial p} = -\frac{\partial\phi}{\partial\lambda} + a\cos\varphi\cdot F_\lambda \quad . \tag{16}$$

Define symmetric angular momentum m_S and antisymmetric angular momentum m_A as

$$m_S = a(a\Omega\cos\varphi + u_S)\cos\varphi \quad , \tag{17}$$

$$m_A = au_A\cos\varphi \quad . \tag{18}$$

Then Eq. (16) can be split into two equations which control the local variation in m_S and that in m_A respectively, namely,

$$\frac{\partial m_S}{\partial t} + \nabla\cdot m_S\boldsymbol{V}_S + \nabla\cdot m_A\boldsymbol{V}_A + \frac{\partial}{\partial p}m_S\omega_S + \frac{\partial}{\partial p}m_A\omega_A$$

$$= -\frac{\partial\phi_S}{\partial\lambda} + a\cos\varphi\cdot(F_\lambda)_S \quad , \tag{19}$$

and

$$\frac{\partial m_A}{\partial t} + \nabla\cdot m_S\boldsymbol{V}_A + \nabla\cdot m_A\boldsymbol{V}_S + \frac{\partial}{\partial p}m_S\omega_A + \frac{\partial}{\partial p}m_A\omega_S$$

$$= -\frac{\partial\phi_A}{\partial\lambda} + a\cos\varphi\cdot(F_\lambda)_A \quad . \tag{20}$$

Then, under the vertical boundary conditions at $p=0$ and p_0 (surface pressure)

$$\omega = 0 \quad , \tag{21}$$

the equations

$$\frac{\partial M_S}{\partial t} = -\frac{1}{g}\int_G \frac{\partial\phi_S}{\partial\lambda}dM + \frac{1}{g}\int_G a\cos\varphi\cdot(F_\lambda)_S dM \quad , \tag{22}$$

and

$$\frac{\partial M_A}{\partial t} = 0 \quad , \tag{23}$$

can be derived. Hence

$$\frac{\partial\hat{M}}{\partial t} = \frac{\partial M_S}{\partial t} \quad , \tag{24}$$

where $\hat{M} = M_S + M_A$; $\frac{1}{g}\int_G(\quad)dM$ represents the integral of the entire atmosphere,

$$M_S = \frac{1}{g}\int_G m_S dM \quad ,$$

$$M_A = \frac{1}{g}\int_G m_A dM \quad .$$

The above equations show that only the zonal differences of the symmetric components of ϕ caused by orography and the symmetric components of F_A contribute to the time variation in the global total angular momentum; and the global total m_A always vanishes.

IV. TIME VARIATION IN VORTICITY

Making curl operator on Eqs. (1) and (2), we can obtain the vorticity equation controlling symmetric motion and that controlling antisymmetric motion.

(1) *Symmetric motion*

$$\frac{\partial \zeta_A}{\partial t}+\boldsymbol{V}_S\cdot\nabla\eta_A+\boldsymbol{V}_A\cdot\nabla\zeta_S+\omega_S\frac{\partial\eta_A}{\partial p}+\omega_A\frac{\partial\zeta_S}{\partial p}+\zeta_S\nabla\cdot\boldsymbol{V}_A$$
$$+\eta_A\nabla\cdot\boldsymbol{V}_S+k\cdot\left(\nabla\omega_S\times\frac{\partial\boldsymbol{V}_S}{\partial p}+\nabla\omega_A\times\frac{\partial\boldsymbol{V}_A}{\partial p}\right)$$
$$=-g\boldsymbol{k}\cdot\nabla\times\frac{\partial\tau_S}{\partial p}\quad. \tag{25}$$

(2) *Antisymmetric motion*

$$\frac{\partial \zeta_S}{\partial t}+\boldsymbol{V}_S\cdot\nabla\zeta_S+\boldsymbol{V}_A\cdot\nabla\eta_A+\omega_S\frac{\partial\zeta_S}{\partial p}+\omega_A\frac{\partial\eta_A}{\partial p}+\zeta_S\nabla\cdot\boldsymbol{V}_S$$
$$+\eta_A\nabla\cdot\boldsymbol{V}_A+\boldsymbol{k}\cdot\left(\nabla\omega_S\times\frac{\partial\boldsymbol{V}_A}{\partial p}+\nabla\omega_A\times\frac{\partial\boldsymbol{V}_S}{\partial p}\right)$$
$$=-g\boldsymbol{k}\cdot\nabla\times\frac{\partial\tau_A}{\partial p}, \tag{26}$$

where ζ is the vertical component of relative vorticity; $\eta=\zeta+f$, absolute vorticity; τ frictional stress; the others are all the symbols conventionally used in meteorology.

It should be noted that we have already taken

$$\boldsymbol{F}=-g\partial\tau/\partial p\quad,$$
$$\zeta_A=\boldsymbol{k}\cdot\nabla\times\boldsymbol{V}_S\quad,$$

and

$$\zeta_S=\boldsymbol{k}\cdot\nabla\times\boldsymbol{V}_A.$$

It will be seen that like the case of equations of motion, the symmetric motion and the antisymmetric motion characterized by the corresponding vorticity equations are still mutually interacted and restricted.

Eqs. (25) and (26) can also be written in the form

$$\frac{\partial\zeta_A}{\partial t}==\nabla\cdot\left(\eta_A\boldsymbol{V}_S+\zeta_S\boldsymbol{V}_A+\omega_S\frac{\partial\boldsymbol{V}_S}{\partial p}\times\boldsymbol{k}+\omega_A\frac{\partial\boldsymbol{V}_A}{\partial p}\times\boldsymbol{k}-g\boldsymbol{k}\times\frac{\partial\tau_S}{\partial p}\right), \tag{27}$$

and

$$\frac{\partial\zeta_S}{\partial t}=-\nabla\cdot\left(\zeta_S\boldsymbol{V}_S+\eta_A\boldsymbol{V}_A+\omega_S\frac{\partial\boldsymbol{V}_A}{\partial p}\times\boldsymbol{k}+\omega_A\frac{\partial\boldsymbol{V}_S}{\partial p}\times\boldsymbol{k}-g\boldsymbol{k}\times\frac{\partial\tau_A}{\partial p}\right), \tag{28}$$

respectively. Hence, at any isobaric surface without intersection with the earth surface we have

$$\frac{\partial Z_S}{\partial t}=0 \quad , \tag{29}$$

and

$$\frac{\partial Z_A}{\partial t}=0 \quad , \tag{30}$$

where Z_S and Z_A are the integrals taking ζ_S and η (or ζ_A) as the integrands over the entire spherical surface respectively.

Eq. (28) indicates that the conservation of Z_S or Z_A still holds if a certain term in the bracket on its right hand side is neglected. Therefore, it is convenient to simplifying the vorticity equation for describing various scale symmetric motions or antisymmetric motions appropriately on the basis of comparing the order of magnitude of its terms.

V. ZONAL MEAN MOTION AND PERTURBED MOTION

1. *The Equations Controlling Zonal Mean Motion and Perturbed Motion*

Separate the related scalar quantities and vector quantities into zonal mean components and perturbed components, namely,

$$M=\overline{M}+M' \quad , \tag{31}$$

$$\boldsymbol{V}=\overline{\boldsymbol{V}}+\boldsymbol{V}' \quad , \tag{32}$$

and

$$\boldsymbol{F}=\overline{\boldsymbol{F}}+\boldsymbol{F}' \quad , \tag{33}$$

where $(\overline{\quad})$ stands for zonal mean quantity, $(\quad)'$ perturbed quantity. The former can be formulated as

$$\overline{(\quad)}=\frac{1}{2\pi}\int_0^{2\pi}(\quad)\,d\lambda \quad . \tag{34}$$

Making zonal mean operations on Eqs. (1)—(4) and (5)—(8) respectively, gives the equa-tions controlling zonal mean motion

$$\begin{aligned}\frac{\partial \overline{\boldsymbol{V}}_S}{\partial t}=&-(\overline{\boldsymbol{V}}_S\cdot\nabla)\overline{\boldsymbol{V}}_S-(\overline{\boldsymbol{V}}_A\cdot\nabla)\overline{\boldsymbol{V}}_A-\overline{(\boldsymbol{V}'_S\cdot\nabla)\boldsymbol{V}'_S}-\overline{(\boldsymbol{V}'_A\cdot\nabla)\boldsymbol{V}'_A}\\&-\overline{\boldsymbol{\omega}}_S\frac{\partial\overline{\boldsymbol{V}}_S}{\partial p}-\overline{\boldsymbol{\omega}}_A\frac{\partial\overline{\boldsymbol{V}}_A}{\partial p}-\overline{\omega'_S\frac{\partial\boldsymbol{V}'_S}{\partial p}}-\overline{\omega'_A\frac{\partial\boldsymbol{V}'_A}{\partial p}}\\&-2\boldsymbol{\Omega}\times\overline{\boldsymbol{V}}_S-\nabla\overline{\phi}_S-\overline{\boldsymbol{F}}_S \quad ,\end{aligned} \tag{35}$$

$$\nabla\cdot\overline{\boldsymbol{V}}_S+\frac{\partial\overline{\boldsymbol{\omega}}_S}{\partial p}=0 \quad , \tag{36}$$

$$\begin{aligned}\frac{\partial\overline{T}_S}{\partial t}=&-\overline{\boldsymbol{V}}_S\cdot\nabla\overline{T}_S-\overline{\boldsymbol{V}}_A\cdot\nabla\overline{T}_A-\overline{\boldsymbol{V}'_S\cdot\nabla T'_S}-\overline{\boldsymbol{V}'_A\cdot\nabla T'_A}-\overline{\boldsymbol{\omega}}_S\frac{\partial\overline{T}_S}{\partial p}\\&-\overline{\boldsymbol{\omega}}_A\frac{\partial\overline{T}_A}{\partial p}-\overline{\omega'_S\frac{\partial T'_S}{\partial p}}-\overline{\omega'_A\frac{\partial T'_A}{\partial p}}+\frac{R}{c_p p}(\overline{T}_S\overline{\boldsymbol{\omega}}_S\\&+\overline{T}_A\overline{\boldsymbol{\omega}}_A+\overline{T'_S\omega'_S}+\overline{T'_A\omega'_A})+\frac{\overline{q}_S}{c_p} \quad ,\end{aligned} \tag{37}$$

$$\frac{\partial\bar{\phi}_S}{\partial p}=-\frac{R\bar{T}_S}{p} \quad , \tag{38}$$

and

$$\frac{\partial\bar{V}_A}{\partial t}=-(\bar{V}_S\cdot\nabla)\,\bar{V}_A-(\bar{V}_A\cdot\nabla)\,\bar{V}_S-\overline{(V'_S\cdot\nabla)V'_A}-\overline{(V'_A\cdot\nabla)V'_S}$$

$$-\bar{\omega}_S\frac{\partial\bar{V}_A}{\partial p}-\bar{\omega}_A\frac{\partial\bar{V}_S}{\partial p}-\overline{\omega'_S\frac{\partial V'_A}{\partial p}}-\overline{\omega'_A\frac{\partial V'_S}{\partial p}}$$

$$-2\boldsymbol{\Omega}\times\bar{V}_A-\nabla\bar{\phi}_A+\bar{F}_A \quad , \tag{39}$$

$$\nabla\cdot\bar{V}_A+\frac{\partial\bar{\omega}_A}{\partial p}=0 \quad , \tag{40}$$

$$\frac{\partial\bar{T}_A}{\partial t}=-\bar{V}_S\cdot\nabla\bar{T}_A-\bar{V}_A\cdot\nabla\bar{T}_S-\overline{V'_S\cdot\nabla T'_A}-\overline{V'_A\cdot\nabla T'_S}$$

$$-\bar{\omega}_S\frac{\partial\bar{T}_A}{\partial p}-\bar{\omega}_A\frac{\partial\bar{T}_S}{\partial p}-\overline{\omega'_S\frac{\partial T'_A}{\partial p}}-\overline{\omega'_A\frac{\partial T'_S}{\partial p}}$$

$$+\frac{R}{c_p p}\left(\bar{T}_S\bar{\omega}_A+\bar{T}_A\bar{\omega}_S+\overline{T'_S\omega'_A}+\overline{T'_A\omega'_S}\right)+\frac{\bar{q}_A}{c_p} \quad , \tag{41}$$

$$\frac{\partial\bar{\phi}_A}{\partial p}=-\frac{R\bar{T}_A}{p} \quad . \tag{42}$$

From Eqs. (1)—(4) and (35)—(38), and Eqs. (5)—(8) and (39)—(42) the equations controlling perturbed motion are written as

$$\frac{\partial V'_S}{\partial t}=-(\bar{V}_S\cdot\nabla)V'_S-(V'_S\cdot\nabla)\,\bar{V}_S-(V'_S\cdot\nabla)V'_S+\overline{(V'_S\cdot\nabla)V'_S}$$

$$-(\bar{V}_A\cdot\nabla)V'_A-(V'_A\cdot\nabla)\,\bar{V}_A-(V'_A\cdot\nabla)V'_A+\overline{(V'_A\cdot\nabla)V'_A}$$

$$-\bar{\omega}_S\frac{\partial V'_S}{\partial p}-\omega'_S\frac{\partial\bar{V}_S}{\partial p}-\omega'_S\frac{\partial V'_S}{\partial p}+\overline{\omega'_S\frac{\partial V'_S}{\partial p}}$$

$$-\bar{\omega}_A\frac{\partial V'_A}{\partial p}-\omega'_A\frac{\partial\bar{V}_A}{\partial p}-\omega'_A\frac{\partial V'_A}{\partial p}+\overline{\omega'_A\frac{\partial V'_A}{\partial p}}$$

$$-2\boldsymbol{\Omega}\times V'_S-\nabla\phi'_S+F'_S \quad , \tag{43}$$

$$\nabla V'_S+\frac{\partial w'_S}{\partial p}=0. \tag{44}$$

$$\frac{\partial T'_S}{\partial t}=-\bar{V}_S\cdot\nabla T'_S-V'_S\cdot\nabla\bar{T}_S-V'_S\cdot\nabla T'_S+\overline{V'_S\cdot\nabla T'_S}$$

$$-\bar{\omega}_S\frac{\partial T'_S}{\partial p}-\omega'_S\frac{\partial\bar{T}_S}{\partial p}-\omega'_S\frac{\partial T'_S}{\partial p}+\overline{\omega'_S\frac{\partial T'_S}{\partial p}}$$

$$-\bar{V}_A\cdot\nabla T'_A-V'_A\cdot\nabla\bar{T}_A-V'_A\cdot\nabla T'_A+\overline{V'_A\cdot\nabla T'_A}$$

$$-\bar{\omega}_A\frac{\partial T'_A}{\partial p}-\omega'_A\frac{\partial \bar{T}_A}{\partial p}-\omega'_A\frac{\partial T'_A}{\partial p}+\overline{\omega'_A\frac{\partial T'_A}{\partial p}}$$

$$+\frac{R}{c_p p}(\bar{T}_S\omega'_S+T'_S\bar{\omega}_S+T'_S\omega'_S-\overline{T'_S\omega'_S}$$

$$+\bar{T}_A\omega'_A+T'_A\bar{\omega}_A+T'_A\omega'_A-\overline{T'_A\omega'_A})+\frac{q'_S}{c_p}\quad, \tag{45}$$

$$\frac{\partial\phi'_S}{\partial p}=-\frac{RT'_S}{p}\quad, \tag{46}$$

and

$$\frac{\partial \boldsymbol{V}'_A}{\partial t}=-(\bar{\boldsymbol{V}}_S\cdot\nabla)\boldsymbol{V}'_A-(\boldsymbol{V}'_S\cdot\nabla)\bar{\boldsymbol{V}}_A-(\boldsymbol{V}'_S\cdot\nabla)\boldsymbol{V}'_A+\overline{(\boldsymbol{V}'_S\cdot\nabla)\boldsymbol{V}'_A}$$

$$-(\bar{\boldsymbol{V}}_A\cdot\nabla)\boldsymbol{V}'_S-(\boldsymbol{V}'_A\cdot\nabla)\bar{\boldsymbol{V}}_S-(\boldsymbol{V}'_A\cdot\nabla)\boldsymbol{V}'_S+\overline{(\boldsymbol{V}'_A\cdot\nabla)\boldsymbol{V}'_S}$$

$$-\bar{\omega}_S\frac{\partial \boldsymbol{V}'_A}{\partial p}-\omega'_S\frac{\partial \bar{\boldsymbol{V}}_A}{\partial p}-\omega'_S\frac{\partial \boldsymbol{V}'_A}{\partial p}+\overline{\omega'_S\frac{\partial \boldsymbol{V}'_A}{\partial p}}$$

$$-\bar{\omega}_A\frac{\partial \boldsymbol{V}'_S}{\partial p}-\omega'_A\frac{\partial \bar{\boldsymbol{V}}_S}{\partial p}-\omega'_A\frac{\partial \boldsymbol{V}'_S}{\partial p}+\overline{\omega'_A\frac{\partial \boldsymbol{V}'_S}{\partial p}}$$

$$-2\boldsymbol{\Omega}\times\boldsymbol{V}'_A-\nabla\phi'_A+\boldsymbol{F}'_A, \tag{47}$$

$$\nabla\cdot\boldsymbol{V}'_A+\frac{\partial\omega'_A}{\partial p}=0, \tag{48}$$

$$\frac{\partial T'_A}{\partial t}=-\bar{\boldsymbol{V}}_S\cdot\nabla T'_A-\boldsymbol{V}'_S\cdot\nabla\bar{T}_A-\boldsymbol{V}'_S\cdot\nabla T'_A+\overline{\boldsymbol{V}'_S\cdot\nabla T'_A}$$

$$-\bar{\boldsymbol{V}}_A\cdot\nabla T'_S-\boldsymbol{V}'_A\cdot\nabla\bar{T}_S-\boldsymbol{V}'_A\cdot\nabla T'_S+\overline{\boldsymbol{V}'_A\cdot\nabla T'_S}$$

$$-\bar{\omega}_S\frac{\partial T'_A}{\partial p}-\omega'_S\frac{\partial \bar{T}_A}{\partial p}-\omega'_S\frac{\partial T'_A}{\partial p}+\overline{\omega'_S\frac{\partial T'_A}{\partial p}}$$

$$-\bar{\omega}_A\frac{\partial T'_S}{\partial p}-\omega'_A\frac{\partial \bar{T}_S}{\partial p}-\omega'_A\frac{\partial T'_S}{\partial p}+\overline{\omega'_A\frac{\partial T'_S}{\partial p}}$$

$$+\frac{R}{c_p p}(\bar{T}_S\omega'_A+T'_S\bar{\omega}_A+\bar{T}_A\omega'_S+T'_A\bar{\omega}_S+T'_S\omega'_A$$

$$+T_A\omega'_S-\overline{T'_S\omega'_A}-\overline{T'_A\omega'_S})+\frac{q'_A}{c_p}, \tag{49}$$

$$\frac{\partial\phi'_A}{\partial p}=-\frac{RT'_A}{p} \tag{50}$$

can also be obtained.

2. *Mutual Conversion between Zonal Mean Energy and Perturbed Energy*

Forming the scalar product of Eq. (35) and $\boldsymbol{V}_S$, and of Eq. (39) and $\boldsymbol{V}_A$ gives

$$\frac{\partial \bar{K}_S}{\partial t}=-C(\bar{K}_S,\bar{K}_A)-C(\bar{K}_S,K'_S)-C(\bar{K}_S,\bar{P}_S)+D(\bar{K}_S) \tag{51}$$

and

$$\frac{\partial \bar{K}_A}{\partial t}=C(\bar{K}_S,\bar{K}_A)-C(\bar{K}_A,K_A')-C(\bar{K}_A,\bar{P}_S)+D(\bar{K}_A), \tag{52}$$

where

$$\bar{K}_S=\frac{1}{g}\int_G \frac{1}{2}\bar{\boldsymbol{V}}_S\cdot\bar{\boldsymbol{V}}_S dM;\ \bar{K}_A=\frac{1}{g}\int_G \frac{1}{2}\bar{\boldsymbol{V}}_A\cdot\bar{\boldsymbol{V}}_A dM,$$

$$C(\bar{K}_S,\bar{K}_A)=\frac{1}{g}\int_G \bar{\boldsymbol{V}}_S\cdot\left[(\bar{\boldsymbol{V}}_A\cdot\nabla)\bar{\boldsymbol{V}}_A+\bar{\omega}_A\frac{\partial\bar{\boldsymbol{V}}_A}{\partial P}\right]dM,$$

$$C(\bar{K}_S,K_S')=\frac{1}{g}\int_G \bar{\boldsymbol{V}}_S\cdot\left[\overline{(\boldsymbol{V}_S'\cdot\nabla)\boldsymbol{V}_S'}+\overline{(\boldsymbol{V}_A'\cdot\nabla)\boldsymbol{V}_A'}\right.$$

$$\left.+\overline{\omega_S'\frac{\partial\boldsymbol{V}_S'}{\partial p}}+\overline{\omega_A'\frac{\partial\boldsymbol{V}_A'}{\partial p}}\right]dM,$$

$$C(\bar{K}_A,K_A')=\frac{1}{g}\int_G \bar{\boldsymbol{V}}_A\cdot\left[\overline{(\boldsymbol{V}_S'\cdot\nabla)\boldsymbol{V}_A'}+\overline{(\boldsymbol{V}_A'\cdot\nabla)\boldsymbol{V}_S'}\right.$$

$$\left.+\overline{\omega_S'\frac{\partial\boldsymbol{V}_A'}{\partial p}}+\overline{\omega_A'\frac{\partial\boldsymbol{V}_S'}{\partial p}}\right]dM,$$

$$C(\bar{K}_S,\bar{P}_S)=\frac{1}{g}\int_G \frac{R}{c_p p}\bar{\boldsymbol{T}}_S\bar{\boldsymbol{\omega}}_S dM \quad ,$$

$$C(\bar{K}_A,\bar{P}_S)=\frac{1}{g}\int_G \frac{R}{C_p p}\bar{\boldsymbol{T}}_A\bar{\boldsymbol{\omega}}_A dM \quad ,$$

$$D(\bar{K}_S)=\frac{1}{g}\int_G \bar{\boldsymbol{V}}_S\cdot\bar{\boldsymbol{F}}_S dM \quad ,$$

$$D(\bar{K}_A)=\frac{1}{g}\int_G \bar{\boldsymbol{V}}_A\cdot\bar{\boldsymbol{F}}_A dM \quad .$$

From Eqs. (37) and (41) it follows

$$\frac{\partial \bar{P}_S}{\partial t}=C(\bar{K}_S,\bar{P}_S)+C(\bar{K}_A,\bar{P}_S)+C(K_S',\bar{P}_S)+C(K_A',\bar{P}_S)+Q(\bar{P}_S) \ , \tag{53}$$

and

$$\frac{\partial \bar{P}_A}{\partial t}=0 \quad , \tag{54}$$

where

$$\bar{P}_S=\frac{1}{g}\int_G c_p\bar{\boldsymbol{T}}_S dM;\ \bar{P}_A=\frac{1}{g}\int_G c_p\bar{\boldsymbol{T}}_A dM \quad ,$$

$$C(K_S',\bar{P}_S)=\frac{1}{g}\int_G \frac{R}{p}T_S'\omega_S' dM \quad ,$$

$$C(K_A',\bar{P}_S)=\frac{1}{g}\int_G \frac{R}{p}T_A'\omega_A' dM \quad ,$$

$$Q(\bar{P}_S)=\frac{1}{g}\int_G \bar{q}_S dM \quad .$$

If we define

$$K'_S=\frac{1}{g}\int_G \frac{1}{2}\boldsymbol{V}'_S\cdot\boldsymbol{V}'_S dM;\quad K'_A=\frac{1}{g}\int_G \frac{1}{2}\boldsymbol{V}'_A\cdot\boldsymbol{V}'_A dM \quad ,$$

$$P'_S=\frac{1}{g}\int_G c_p T'_S dM;\quad P'_A=\frac{1}{g}\int_G c_p T'_A dM \quad ,$$

then

$$\frac{\partial K'_S}{\partial t}=\frac{\partial K_S}{\partial t}-\frac{\partial \bar{K}_S}{\partial t} \quad ,$$

$$\frac{\partial K'_A}{\partial t}=\frac{\partial K_A}{\partial t_A}-\frac{\partial \bar{K}_A}{\partial t} \quad ,$$

where

$$K_S=\frac{1}{g}\int_G \frac{1}{2}\boldsymbol{V}_S\cdot\boldsymbol{V}_S dM;\quad K_A=\frac{1}{g}\int_G \frac{1}{2}\boldsymbol{V}_A\cdot\boldsymbol{V}_A dM \quad .$$

Then, from Eqs. (1), (5), (51) and (52) we obtain

$$\frac{\partial K'_S}{\partial t}=C(\bar{K}_S, K'_S)-C(K'_S, K'_A)-C(K'_S, \bar{P}_S)+D(K'_S) \quad , \tag{55}$$

$$\frac{\partial K'_A}{\partial t}=C(\bar{K}_A, K'_A)+C(K'_S, K'_A)-C(K'_A, \bar{P}_S)+D(K'_A) \quad , \tag{56}$$

where

$$C(K'_S, K'_A)=\frac{1}{g}\int_G\Big\{\bar{\boldsymbol{V}}_S\cdot\Big[(\boldsymbol{V}'_A\cdot\nabla)\boldsymbol{V}'_A+\omega'_A\frac{\partial \boldsymbol{V}'_A}{\partial p}\Big]+\boldsymbol{V}'_S\cdot\Big[(\boldsymbol{V}'_A\cdot\nabla)\bar{\boldsymbol{V}}_A$$

$$+(\bar{\boldsymbol{V}}_A\cdot\nabla)\boldsymbol{V}'_A+(\boldsymbol{V}'_A\cdot\nabla)\boldsymbol{V}'_A+\bar{\boldsymbol{\omega}}_A\frac{\partial \boldsymbol{V}'_A}{\partial p}+\omega'_A\frac{\partial \bar{\boldsymbol{V}}'_A}{\partial p}+\omega'_A\frac{\partial \boldsymbol{V}'_A}{\partial p}\Big]\Big\}dM \; ,$$

$$D(K'_S)=\frac{1}{g}\int_G \boldsymbol{V}'_S\cdot\boldsymbol{F}'_S dM;\quad D(K'_A)=\frac{1}{g}\int_G \boldsymbol{V}'_A\cdot\boldsymbol{F}'_A dM \quad .$$

It will readily be seen that

$$\frac{\partial P'_S}{\partial t}=0 \quad , \tag{57}$$

and

$$\frac{\partial P'_A}{\partial t}=0 \quad . \tag{58}$$

From Eqs. (51)—(58) the transformation relationships between various forms of zonal mean energy and of perturbed energy can be seen. They are illustrated in Fig. 1.

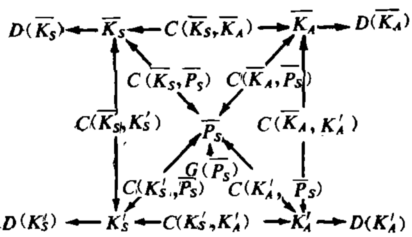

Fig. 1. Conversions between various forms of zonal mean energy and perturbed energy.

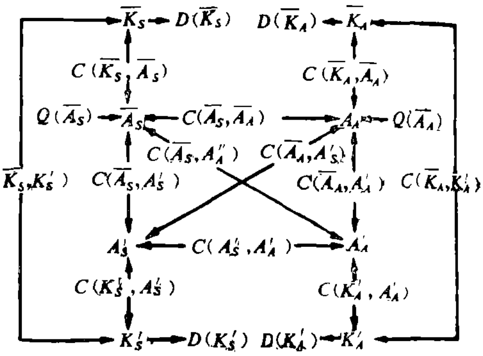

Fig. 2. Conversions between varicus forms of energy.

$A \leftarrow C(A,B) \rightarrow B$ stands for that A and B convert to each other through $C(A,B)$; $D(A) \leftarrow A$, A is dissipated through $D(A)$: $Q(A) \rightarrow A$, A is produced through $Q(A)$.

3. *Available Potential Energy and Energy Conversions*

Dividing the thermodynamic equation by the temperature T, and then replacing it by the global mean temperature $[T]$ and letting $T=[T]+T^*$, we have

$$\frac{1}{[T]}\left(\frac{\partial T^*}{\partial t}+\boldsymbol{V}\cdot\nabla T^*+\omega\frac{\partial T^*}{\partial p}+\omega\frac{\partial [T]}{\partial p}-\left[\omega\frac{\partial T^*}{\partial p}\right]\right)$$

$$=\frac{R}{c_p p}\omega+\frac{q^*}{c_p[T]} \quad . \tag{59}$$

Utilizing Eq. (9) and separating T^* into T_S^* and T_A^*, we obtain from Eq. (59)

$$\frac{\partial T_S^*}{\partial t}+\boldsymbol{V}_S\cdot\nabla T_S^*+\boldsymbol{V}_A\cdot\nabla T_A^*+\omega_S\frac{\partial T_S^*}{\partial p}+\omega_A\frac{\partial T_A^*}{\partial p}+\omega_S\frac{\partial [T]}{\partial p}$$

$$=\left[\omega\frac{\partial T^*}{\partial p}\right]+\frac{R[T]}{c_p p}\omega_S+\frac{q_S^*}{c_p}, \tag{60}$$

and

$$\frac{\partial T_A^*}{\partial t}+\boldsymbol{V}_S\cdot\nabla T_A^*+\boldsymbol{V}_A\cdot\nabla T_S^*+\omega_S\frac{\partial T_A^*}{\partial p}+\omega_A\frac{\partial T_S^*}{\partial p}+\omega_A\frac{\partial [T]}{\partial p}$$

$$=\frac{R[T]}{c_p p}\omega_A+\frac{q_A^*}{c_p} \quad , \tag{61}$$

where $q^*=q_S^*+q_A^*$.

Define

$$A_S=\frac{1}{2g}\gamma c_p\int_G T_S^{*2}dM \quad , \tag{62}$$

$$A_A=\frac{1}{2g}\gamma c_p\int_G T_A^{*2}dM \quad , \tag{63}$$

where $\gamma=\left([T]-pc_pR^{-1}\dfrac{\partial[T]}{\partial p}\right)^{-1}$.

Multiplying Eqs. (60),(61) by T_S^* and T_A^* respectively and noting that

$$\int_G T_S\omega_S dM=\int_G T_S^*\omega_S dM \quad ,$$

give

$$\frac{\partial A_S}{\partial t}=-C(A_S,A_A)+C(K_S,A_S)+Q(A_S) \quad , \tag{64}$$

and

$$\frac{\partial A_A}{\partial t}=C(A_S,A_A)+C(K_A,A_A)+Q(A_A) \quad , \tag{65}$$

where

$$C(A_S,A_A)=\frac{c_p\gamma}{g}\int_G T_S^*\left(\boldsymbol{V}_A\cdot\nabla T_A^*+\omega_A\frac{\partial T_A^*}{\partial p}\right)dM \quad ,$$

$$C(K_S,A_S)=\frac{1}{g}\int_G\frac{R}{p}T_S^*\omega_S dM=\frac{1}{g}\int_G\frac{R}{p}T_S\omega_S dM \quad ,$$

$$Q(A_S)=\frac{\gamma}{g}\int_G T_S^*q_S^*dM \quad ,$$

$$C(K_A,A_A)=\frac{1}{g}\int_G\frac{R}{p}T_A^*\omega_A dM=\frac{1}{g}\int_G\frac{R}{p}T_A\omega_A dM \quad ,$$

$$Q(A_A)=\frac{\gamma}{g}\int_G T_A^*q_A^*dM \quad .$$

Let

$$T^*=\hat{T}+T'',\hat{T}=\overline{T}^* \quad , \tag{66}$$

and define

$$\overline{A}_S=\frac{1}{2g}\gamma c_p\int_G\hat{T}_S^2dM \quad , \tag{67}$$

$$\overline{A}_A=\frac{1}{2g}\gamma c_p\int_G\hat{T}_A^2dM \quad , \tag{68}$$

$$A_S'=\frac{1}{2g}\gamma c_p\int_G T_S''^2dM , \tag{69}$$

$$A_A'=\frac{1}{2g}\gamma c_p\int_G T_A''^2dM \quad . \tag{70}$$

Then taking zonal mean operation on Eq. (60) and Eq. (61) and multiplying them by $\hat{T}_S$ and $\hat{T}_A$ respectively, we obtain

$$\frac{\partial\overline{A}_S}{\partial t}=-C(\overline{A}_S,\overline{A}_A)-C(\overline{A}_S,A_S')-C(\overline{A}_S,A_A')$$
$$+C(\overline{K}_S,\overline{A}_S)+Q(\overline{A}_S) \tag{71}$$

and

$$\frac{\partial \bar{A}_A}{\partial t}=C(\bar{A}_S,\bar{A}_A)-C(\bar{A}_A,A_S')-C(\bar{A}_A,A_A')$$
$$+C(\bar{K}_A,\bar{A}_A)+Q(\bar{A}_A) \quad . \tag{72}$$

Subtraction of (71) and (72) from (64) and (65) respectively gives

$$\frac{\partial A_S'}{\partial t}=C(\bar{A}_S,A_S')+C(\bar{A}_A,A_S')-C(A_S',A_A')$$
$$+C(K_S',A_S')+Q(A_S') \quad , \tag{73}$$

$$\frac{\partial A_A'}{\partial t}=C(\bar{A}_S,A_A')+C(\bar{A}_A,A_A')+C(A_S',A_A') \tag{74}$$
$$+C(K_A',A_A')+Q(A_A')$$

where

$$C(\bar{A}_S,\bar{A}_A)=\frac{\gamma c_p}{g}\int_G \hat{T}_S\left(\bar{\boldsymbol{V}}_A\cdot\nabla\hat{T}_A+\bar{\omega}_A\frac{\partial\hat{T}_A}{\partial p}\right)dM \ ,$$

$$C(\bar{A}_S,A_S')=\frac{\gamma c_p}{g}\int_G \hat{T}_S\left(\overline{\boldsymbol{V}_S'\cdot\nabla T_S''+\omega_S'\frac{\partial T_S''}{\partial p}}\right)dM \ ,$$

$$C(\bar{A}_S,A_A')=\frac{\gamma c_p}{g}\int_G \hat{T}_S\left(\overline{\boldsymbol{V}_S'\cdot\nabla T_A''+\omega_A'\frac{\partial T_A''}{\partial p}}dM \ ,$$

$$C(\bar{A}_A,A_S')=\frac{\gamma c_p}{g}\int_G \hat{T}_A\left(\overline{\boldsymbol{V}_S'\cdot\nabla T_S''+\omega_A'\frac{\partial T_S''}{\partial p}}\right)dM \ ,$$

$$C(\bar{A}_A,A_A')=\frac{\gamma c_p}{g}\int_G \hat{T}_A\left(\overline{\boldsymbol{V}_S'\cdot\nabla T_A''+\omega_S'\frac{\partial T_A''}{\partial p}}\right)dM \ ,$$

$$C(\bar{K}_S,\bar{A}_S)=\frac{1}{g}\int_G\frac{R}{p}\hat{T}_S\omega_S dM,$$

$$C(\bar{K}_A,\bar{A}_A)=\frac{1}{g}\int_G\frac{R}{p}\hat{T}_A\bar{\omega}_A dM,$$

$$C(A_S',A_A')=\frac{\gamma c_p}{g}\int_G T_S''\left(\bar{\boldsymbol{V}}_A\cdot\nabla T_A''+\boldsymbol{V}_A'\cdot\nabla T_A''+\bar{\omega}_A\frac{\partial T_A''}{\partial p}+\omega_A'\frac{\partial T_A''}{\partial p}\right)dM,$$

$$Q(\bar{A}_S)=\frac{\gamma}{g}\int_G\hat{T}_S\hat{q}_S dM \ ,$$

$$Q(\bar{A}_A)=\frac{\gamma}{g}\int_G\hat{T}_A\hat{q}_A dM \ ,$$

$$C(K_S',A_S')=\frac{1}{g}\int_G\frac{R}{p}T_S''\omega_S' dM,$$

$$C(K_A',A_A')=\frac{1}{g}\int_G\frac{R}{p}T_A''\omega_A' dM,$$

$$Q(A'_S)=\frac{\gamma}{g}\int_G T''_S q'_S dM,$$

$$Q(A'_A)=\frac{\gamma}{g}\int_G T''_A q'_A dM.$$

It can be seen from the above expressions that the conversion terms $C(\overline{K}_S,\overline{P}_S)$, $C(\overline{K}_A,\overline{P}_S)$, $C(K'_S,\overline{P}_S)$, $C(K'_A,\overline{P}_S)$ in Eqs. (51), (52) and Eqs. (55), (56) are the same as $C(\overline{K}_S,\overline{A}_S)$, $C(\overline{K}_A,\overline{A}_A)$, $C(K'_S,A'_S)$, $C(K'_A,A'_A)$ in Eqs. (71)—(74)respectively. Then it follows

$$\frac{\partial \overline{K}_S}{\partial t}=-C(\overline{K}_S,\overline{K}_A)-C(\overline{K}_S,K'_S)-C(\overline{K}_S,\overline{A}_S)+D(\overline{K}_S), \tag{75}$$

$$\frac{\partial \overline{K}_A}{\partial t}=C(\overline{K}_S,\overline{K}_A)-C(\overline{K}_A,K'_A)-C(\overline{K}_A,\overline{A}_A)+D(\overline{K}_A) \quad , \tag{76}$$

$$\frac{\partial K'_S}{\partial t}=C(\overline{K}_S,K'_S)-C(K'_S,K'_A)-C(K'_S,A'_S)+D(K'_S) \quad , \tag{77}$$

$$\frac{\partial K'_A}{\partial t}=C(\overline{K}_A,K'_A)+C(K'_S,K'_A)-C(K'_A,A'_A)+D(K'_A) \quad . \tag{78}$$

The above equations indicate that in addition to the conversions between available potential energy and kinetic energy, at the present case there are still conversions between symmetric energy (including available potential energy and kinetic energy) and antisymmetric energy, and cross conversions between zonal mean energy and perturbed energy such as $C(\overline{A}_S,A'_A)$ and $C(\overline{A}_A,A'_S)$ and so on. The conversions between various forms of energy can be illustrated in Fig. 2. It shows that the energy conversions are much more complicated than those cases without separation of the motion into symmetric and antisymmetric components, and much more complicated than the other cases in this paper.

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