SYMMETRIC AND ASYMMETRIC MOTIONS IN THE BAROTROPIC FILTERED MODEL ATMOSPHERE

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ABSTRACT

By expressing the stream function in terms of the symmetric and the antisymmetric components with respect to the equator, the barotropic vorticity equation can be separated into two equations, one describing the behaviours of the atmospheric symmetric motion, and the other those of the atmospheric antisymmetric motion. From them, for the entire global surface at the equivalent barotropic level, the conservations of several basic quantities, such as vorticity, angular momentum etc. in the symmetric and antisymmetric cases, have been discussed. In addition, the energy budget equations and the energy conservation expressions for the two kinds of motion are given. It can be seen from them that there are not only the conversion between the zonal mean kinetic energy and the disturbed kinetic energy in the atmosphere, but also the conversion between the kinetic energy generated by the symmetric and antisymmetric motions.

In the case of including orography and horizontal diffusion into the vorticity equation, a mechanism of the generation of asymmetric behaviours of the atmosphere is proposed. The results show that the asymmetric distribution of orography and that of horizontal diffusion coefficients are likely the causes leading to the asymmetric motion. Finally, a theoretical comparison between the global model and the hemispheric one from the physical point of view is made.

I. INTRODUCTION

It is frequently found on global weather or climatological maps that many meteorological elements, such as pressure, temperature or zonal winds, to considerable extent, are symmetrically distributed with the latitude with respect to the equator, especially in spring or autumn seasons. Owing to these phenomena, Serra (1964) even used them as the basis of the global analysis. Of course, the so-called symmetry is not exactly valid, because there still exist asymmetries in some degree in different seasons and regions. In addition, even the zonal distributions of the elements mentioned above are far from being homogeneous. Some meteorologists attributed such asymmetries or inhomogeneities to the differences of season, orography, sea and land contrast, underlying surface state and so on. Therefore, there remain many problems to be tackled, e.g. through what mechanism is the asymmetric motion produced? Why is the symmetric motion repeated periodically in time? Are the wind and the pressure fields symmetric or asymmetric without the differences? These are also the problems the authors want to investigate. This paper is a part of the research.

II. SEPARATION OF THE BAROTROPIC FILTERED VORTICITY EQUATION

The vorticity equation can be written as

$$\frac{\partial \zeta}{\partial t} = -\boldsymbol{V}\cdot\nabla(\zeta+f), \qquad (1)$$

where $\zeta=\nabla^2\psi$, $\boldsymbol{V}=\boldsymbol{k}\times\nabla\psi$, ψ is the stream function, f the Coriolis parameter.

Considering any motion as a combination of the symmetric and antisymmetric components, an asymmetric motion can be described provided the equations governing the components are given.

Any meteorological element $F(\lambda,\varphi,t)$ can be expressed as

$$F(\lambda,\varphi,t)=F_A(\lambda,\varphi,t)+F_S(\lambda,\varphi,t), \tag{2}$$

where the subscripts S and A mean symmetric and antisymmetric, respectively, thus

$$F_A(\lambda,\varphi,t)=-F_A(\lambda,-\varphi,t), \tag{3}$$

and

$$F_S(\lambda,\varphi,t)=F_S(\lambda,-\varphi,t). \tag{4}$$

For example,

$$F_A=\frac{1}{2}[F(\lambda,\varphi,t)-F(\lambda,-\varphi,t)]$$

and

$$F_S=\frac{1}{2}[F(\lambda,\varphi,t)+F(\lambda,-\varphi,t)]$$

may be taken to express F_S and F_A. Thus we have

$$\psi=\psi_A+\psi_S,$$

and

$$\zeta=\zeta_A+\zeta_S,$$

where $\zeta_A=\nabla^2\psi_A$ and $\zeta_S=\nabla^2\psi_S$.

If we define

$$\boldsymbol{V}=\boldsymbol{V}_A+\boldsymbol{V}_S,$$

where $\boldsymbol{V}_A=\boldsymbol{k}\times\nabla\psi_S$ and $\boldsymbol{V}_S=\boldsymbol{k}\times\nabla\psi_A$, then substituting the expressions of ζ and of $\boldsymbol{V}$ into Eq. (1), we can obtain

$$\frac{\partial\zeta_A}{\partial t}+\boldsymbol{V}_S\cdot\nabla\eta_A+\boldsymbol{V}_A\cdot\nabla\zeta_S=-\left(\frac{\partial\zeta_S}{\partial t}+\boldsymbol{V}_S\cdot\nabla\zeta_S+\boldsymbol{V}_A\cdot\nabla\eta_A\right),$$

where $\eta_A=\zeta_A+f$.

In the above equation, the left hand side is antisymmetric and the right hand side symmetric, so they both should vanish, namely

$$\frac{\partial\zeta_A}{\partial t}+\boldsymbol{V}_S\cdot\nabla\eta_A+\boldsymbol{V}_A\cdot\nabla\zeta_S=0, \tag{5}$$

and

$$\frac{\partial\zeta_S}{\partial t}+\boldsymbol{V}_S\cdot\nabla\zeta_S+\boldsymbol{V}_A\cdot\nabla\eta_A=0. \tag{6}$$

III. CONSERVATIONS OF TOTAL VORTICITY AND TOTAL ANGULAR MOMENTUM

1. *Conservation of Total Vorticity*

Since $\nabla\cdot\boldsymbol{V}_S=\nabla\cdot\boldsymbol{V}_A\equiv0$, integrating Eqs. (5) and (6) over the global surface gives

$$\frac{\partial}{\partial t}\int_E\zeta_A dA=\frac{\partial}{\partial t}\int_E\eta_A dA=0, \tag{7}$$

and

$$\frac{\partial}{\partial t}\int_E \zeta_S dA=0, \tag{8}$$

where E represents the global integration, dA the area element.

The above two expressions show that the total ζ_S, ζ_A and η_A are all conservative over the global surface.

2. *Conservation of Total Angular Momentum*

Define

$$M=\int_E a\left(a\cos\varphi\cdot\Omega-\frac{1}{a}\frac{\partial\psi}{\partial\varphi}\right)\cos\varphi dA=M_S+M_A, \tag{9}$$

where

$$M_S=\int_E a\left(a\cos\varphi\cdot\Omega-\frac{1}{a}\frac{\partial\psi_A}{\partial\varphi}\right)\cos\varphi dA,$$

and

$$M_A=-\int_E a\cos\varphi\cdot\frac{1}{a}\frac{\partial\psi_S}{\partial\varphi}dA.$$

Then according to Machenhauer (1979), we have

$$\frac{\partial M_S}{\partial t}=a^2\int_E \mu\frac{\partial\zeta_A}{\partial t}dA, \tag{10}$$

where $\mu=\sin\varphi$. Since

$$\mu(\boldsymbol{V}_S\cdot\nabla\eta_A+\boldsymbol{V}_A\cdot\nabla\zeta_S)=\nabla\cdot\eta_A\mu\boldsymbol{V}_S+\nabla\cdot\zeta_S\mu\boldsymbol{V}_A-\eta_A\nabla\cdot\mu\boldsymbol{V}_S-\zeta_S\nabla\cdot\mu\boldsymbol{V}_A, \tag{11}$$

and

$$\eta_A\nabla\cdot\mu\boldsymbol{V}_S=\frac{1}{a^2}\eta_A\frac{\partial\psi_A}{\partial\lambda}=\frac{1}{a^2}\left(\frac{\partial}{\partial\lambda}\eta_A\psi_A-\nabla\cdot\psi_A\nabla\frac{\partial\psi_A}{\partial\lambda}+\frac{1}{2}\frac{\partial}{\partial\lambda}\nabla\psi_A\cdot\nabla\psi_A\right),$$

$$\zeta_S\nabla\cdot\mu\boldsymbol{V}_A=\frac{1}{a^2}\left(\frac{\partial}{\partial\lambda}\zeta_S\psi_S-\nabla\cdot\psi_S\nabla\frac{\partial\psi_S}{\partial\lambda}+\frac{1}{2}\frac{\partial}{\partial\lambda}\nabla\psi_S\cdot\nabla\psi_S\right),$$

then, from Eqs. (5) and (6), we directly obtain

$$\frac{\partial M_S}{\partial t}=0. \tag{12}$$

Similarly, the expressions

$$\frac{\partial M_A}{\partial t}=a^2\int_E \mu\frac{\partial\zeta_S}{\partial t}dA=0 \tag{13}$$

and

$$\frac{\partial M}{\partial t}=0 \tag{14}$$

can be got. Therefore, the expressions (12), (13) and (14) are consistent with the classical conservation of total angular momentum in the barotropic filtered model atmosphere.

IV. CONVERSION AND CONSERVATION OF ENSTROPHY

Multiplying Eqs. (5) and (6) by η_A and ζ_S respectively yields

$$\frac{\partial}{\partial t}\int_E \eta_A^2 dA = 2\int_E (\zeta_S \boldsymbol{V}_A \cdot \nabla \eta_A) dA, \tag{15}$$

and

$$\frac{\partial}{\partial t}\int_E \zeta_S^2 dA = -2\int_E (\zeta_S \boldsymbol{V}_A \cdot \nabla \eta_A) dA. \tag{16}$$

It indicates that the total symmetric and antisymmetric enstrophies can be converted into each other through the term

$$2\int_E (\zeta_S \boldsymbol{V}_A \cdot \nabla \eta_A) dA.$$

Since

$$\zeta_S \frac{\partial \eta_A}{\partial t} + \eta_A \frac{\partial \zeta_S}{\partial t} = -\nabla \cdot \left(\frac{1}{2}\zeta_S^2 \boldsymbol{V}_A + \frac{1}{2}\eta_A^2 \boldsymbol{V}_A + \eta_A \zeta_S \boldsymbol{V}_S \right), \tag{17}$$

then

$$2\int_E \left(\zeta_S \frac{\partial \eta_A}{\partial t} + \eta_A \frac{\partial \zeta_S}{\partial t} \right) dA = 0. \tag{18}$$

Summing up Eqs. (15), (16) and (18), we have

$$\frac{\partial}{\partial t}\int_E \eta^2 dA = 0, \tag{19}$$

where $\eta = \zeta + f$.

V. CONVERSION BETWEEN SYMMETRIC AND ANTISYMMETRIC KINETIC ENERGY

It is known that the total kinetic energy is conservative in the filtered model atmosphere, namely

$$\frac{\partial K}{\partial t} = -\int_E \psi \frac{\partial \zeta}{\partial t} dA = 0, \tag{20}$$

where $K = \int_E \frac{1}{2}(\nabla\psi \cdot \nabla\psi) dA$. Thus

$$\begin{aligned}\frac{\partial K}{\partial t} &= -\int_E \left(\psi_A \frac{\partial \eta_A}{\partial t} + \psi_S \frac{\partial \zeta_S}{\partial t} + \psi_S \frac{\partial \eta_A}{\partial t} + \psi_A \frac{\partial \zeta_S}{\partial t} \right) dA \\ &= \frac{\partial K_A}{\partial t} + \frac{\partial K_S}{\partial t} + \frac{\partial K_{SA}}{\partial t},\end{aligned} \tag{21}$$

where

$$K_S = \int_E \frac{1}{2}(\nabla\psi_A \cdot \nabla\psi_A) dA, \tag{22}$$

$$K_A = \int_E \frac{1}{2}(\nabla\psi_S \cdot \nabla\psi_S) dA, \tag{23}$$

and

$$K_{SA} = \int_E (\nabla\psi_S \cdot \nabla\psi_A) dA. \tag{24}$$

Now we derive the expressions for the time tendency of K_S, K_A and K_{SA}.

Making use of Eqs. (5) and (6) leads to

$$\frac{\partial K_S}{\partial t}=-\int_E \psi_A \frac{\partial \eta_A}{\partial t}dA=-\int_E (\zeta_S \nabla\cdot\psi_A \boldsymbol{V}_A)dA, \tag{25}$$

and

$$\frac{\partial K_A}{\partial t}=-\int_E \psi_S \frac{\partial \zeta_S}{\partial t}dA=-\int_E (\zeta_S \nabla\cdot\psi_S \boldsymbol{V}_S)dA. \tag{26}$$

Since

$$\nabla\cdot(\psi_S \boldsymbol{V}_S)+\nabla\cdot(\psi_A \boldsymbol{V}_A)=\boldsymbol{V}_A\cdot\nabla\psi_A+\boldsymbol{V}_S\cdot\nabla\psi_S=0,$$

then

$$\frac{\partial K_A}{\partial t}=\int_E (\zeta_S \nabla\cdot\psi_A \boldsymbol{V}_A)dA. \tag{27}$$

The expressions (25) and (27) show that the symmetric and antisymmetric kinetic energy can be converted into each other through $\zeta_S\nabla\cdot\psi_A\boldsymbol{V}_A$.

As for K_{SA}, we have

$$\frac{\partial K_{SA}}{\partial t}=\int_E \{\psi_S(\boldsymbol{V}_A\cdot\nabla\zeta_S+\boldsymbol{V}_S\cdot\nabla\eta_A)+\psi_A(\boldsymbol{V}_S\cdot\nabla\zeta_S+\boldsymbol{V}_A\cdot\nabla\eta_A)\}dA$$

$$=\int_E \eta_A\nabla\cdot(\psi_S\boldsymbol{V}_S+\psi_A\boldsymbol{V}_A)dA=0. \tag{28}$$

Hence

$$\frac{\partial}{\partial t}(K_S+K_A)=\frac{\partial K}{\partial t}=0. \tag{29}$$

VI. CONVERSION OF ZONAL MEAN KINETIC ENERGY AND DISTURBED KINETIC ENERGY

Define the zonal mean of any meteorological element F as

$$\bar{F}=\frac{1}{2\pi}\int_0^{2\pi} F d\lambda, \tag{30}$$

and its disturbed quantity F' as

$$F'=F-\bar{F}. \tag{31}$$

Then, from Eqs. (5) and (6) we have

$$\frac{\partial \bar{\zeta}_S}{\partial t}+\bar{\boldsymbol{V}}_S\cdot\nabla\bar{\zeta}_S+\bar{\boldsymbol{V}}_A\cdot\nabla\bar{\eta}_A+\overline{\boldsymbol{V}'_S\cdot\nabla\zeta'_S}+\overline{\boldsymbol{V}'_A\cdot\nabla\eta'_A}=0, \tag{32}$$

$$\frac{\partial \zeta'_S}{\partial t}+\boldsymbol{V}_S\cdot\nabla\zeta_S+\boldsymbol{V}_A\cdot\nabla\eta_A-(\bar{\boldsymbol{V}}_S\cdot\nabla\bar{\zeta}_S+\bar{\boldsymbol{V}}_A\cdot\nabla\bar{\eta}_A+\overline{\boldsymbol{V}'_S\cdot\nabla\zeta'_S}+\overline{\boldsymbol{V}'_A\cdot\nabla\eta'_A})=0, \tag{33}$$

$$\frac{\partial \bar{\zeta}_A}{\partial t}+\bar{\boldsymbol{V}}_S\cdot\nabla\bar{\eta}_A+\bar{\boldsymbol{V}}_A\cdot\nabla\bar{\zeta}_S+\overline{\boldsymbol{V}'_S\cdot\nabla\eta'_A}+\overline{\boldsymbol{V}'_A\cdot\nabla\zeta'_S}=0, \tag{34}$$

and

$$\frac{\partial \zeta'_A}{\partial t}+\boldsymbol{V}_S\cdot\nabla\eta_A+\boldsymbol{V}_A\cdot\nabla\zeta_S-(\bar{\boldsymbol{V}}_S\cdot\nabla\bar{\eta}_A+\bar{\boldsymbol{V}}_A\cdot\nabla\bar{\zeta}_S+\overline{\boldsymbol{V}'_S\cdot\nabla\eta'_A}+\overline{\boldsymbol{V}'_A\cdot\nabla\zeta'_S}=0. \tag{35}$$

Thus

$$\frac{\partial \bar{K}_S}{\partial t} = -C(\bar{K}_S, \bar{K}_A) + C(\bar{K}_S, K'_S), \tag{36}$$

$$\frac{\partial K'_S}{\partial t} = -C(\bar{K}_S, K'_S) + C(K'_S, K'_A), \tag{37}$$

$$\frac{\partial \bar{K}_A}{\partial t} = C(\bar{K}_S, \bar{K}_A) + C(\bar{K}_A, K'_A), \tag{38}$$

and

$$\frac{\partial K'_A}{\partial t} = -C(\bar{K}_A, K'_A) - C(K'_S, K'_A), \tag{39}$$

where

$$C(\bar{K}_S, \bar{K}_A) = \int_E \bar{\zeta}_S (\nabla \cdot \bar{\Phi}_A \overline{V}_A) dA,$$

$$C(\bar{K}_S, K'_S) = \int_E \bar{u}_S \overline{(\zeta'_A v'_A + \zeta'_S v'_S)} dA,$$

$$C(\bar{K}_A, K'_A) = \int_E \bar{u}_A \overline{(\zeta'_S v'_A + \zeta'_A v'_S)} dA,$$

and

$$C(K'_S, K'_A) = \int_E \{\bar{\zeta}_S (\nabla \cdot \bar{\Phi}_A \bar{V}_A) - \zeta_S \nabla \cdot (\psi_A V_A)\} dA.$$

Since

$$\frac{\partial K_S}{\partial t} = \int_E \frac{1}{2} \frac{\partial}{\partial t} (\nabla \bar{\Phi}_A \cdot \nabla \bar{\Phi}_A + \nabla \psi'_A \cdot \nabla \psi'_A) \, dA, \tag{40}$$

then

$$\frac{\partial K_S}{\partial t} = \frac{\partial \bar{K}_S}{\partial t} + \frac{\partial K'_S}{\partial t}. \tag{41}$$

In similar manner

$$\frac{\partial K_A}{\partial t} = \frac{\partial \bar{K}_A}{\partial t} + \frac{\partial K'_A}{\partial t}. \tag{42}$$

Therefore, the conversions between the zonal mean kinetic energy and the disturbed kinetic energy in the cases with and without making separation of motion are quite different from each other. In the case of separation, there exists not only the conversion between the zonal mean

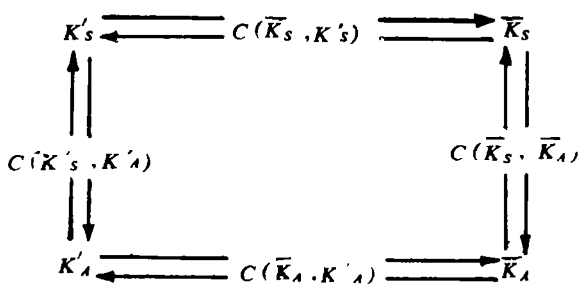

Fig. 1. The conversion relationship among $\bar{K}_S$, $\bar{K}_A$, K'_S and K'_A.

kinetic energy and the disturbed kinetic energy, but also the conversion between $\bar{K}_S$ and $\bar{K}_A$ and that between K'_S and K'_A.

The conversion among $\bar{K}_S$, K'_S, $\bar{K}_A$ and K'_A can be formally illustrated in Fig. 1.

VII. THE GENERATION OF ASYMMETRIC MOTION

1. *The Governing Equation*

As stated in the previous sections, there exist, to some extent, asymmetries in the pressure and temperature fields. It seems that the wind field, which is closely connected with the pressure field, should have certain asymmetries as well. Therefore, we may ask how such asymmetries are produced. It is an interesting question we will deal with.

Consider the vorticity equation in the form

$$\frac{\partial \zeta}{\partial t}+\boldsymbol{V}\cdot\nabla(\zeta+f)=f\frac{\partial \omega}{\partial p}+\mu_1\nabla^2\zeta, \tag{43}$$

where μ_1 is a constant, $\omega=dp/dt$.

Using the difference quotient $(\omega_0-0)/p_0$ to replace $\partial\omega/\partial p$ and employing the following lower boundary condition

$$\omega_0\approx-\rho_0 g w_0=-\frac{p_0 g}{RT_0}\boldsymbol{V}_0\cdot\nabla z_0, \tag{44}$$

we can rewrite Eq. (43) as

$$\frac{\partial \zeta}{\partial t}+\boldsymbol{V}\cdot\nabla(\zeta+f)+\mu_2\boldsymbol{V}\cdot\nabla z_0=\mu_1\nabla^2\zeta, \tag{45}$$

where the subscripts circles denote surface quantities; z_0 the height above sea level; $\mu_2=af/H$, $H=RT_0/g$ and $a=|\boldsymbol{V}_0|/|\boldsymbol{V}|$, the latter two are assumed to be constant. Note that μ_2 is a function of latitude, being antisymmetric with respect to the equator.

2. *Separation of the Equation*

For convenience of discussion, Eq. (45) needs separating into two equations describing the symmetric motion and the antisymmetric motion respectively.

By using the technique of separation and the same notations as in Section II, from Eq. (45), we have

$$\frac{\partial \zeta_A}{\partial t}+\boldsymbol{V}_S\cdot\nabla\eta_A+\boldsymbol{V}_A\cdot\nabla\zeta_S+\mu_2(\boldsymbol{V}_A\cdot\nabla z_{0A}+\boldsymbol{V}_S\cdot\nabla z_{0S})=\mu_1\nabla^2\zeta_A, \tag{46}$$

and

$$\frac{\partial \zeta_S}{\partial t}+\boldsymbol{V}_S\cdot\nabla\zeta_S+\boldsymbol{V}_A\cdot\nabla\eta_A+\mu_2(\boldsymbol{V}_A\cdot\nabla z_{0S}+\boldsymbol{V}_S\cdot\nabla z_{0A})=\mu_1\nabla^2\zeta_S. \tag{47}$$

3. *The Invariance of Symmetric Motion*

Assume

$$\nabla z_{0A}=0, \tag{48}$$

and at $t=t_0$,

$$\psi=\psi_A, \tag{49}$$

namely

$$\psi_S=0. \tag{49'}$$

Then, at $t=t_0$, Eqs. (46) and (47) become

$$\frac{\partial \zeta_A}{\partial t} = -\boldsymbol{V}_S \cdot \nabla \eta_A - \mu_2 \boldsymbol{V}_S \cdot \nabla z_{0S} + \mu_1 \nabla^2 \zeta_A, \tag{50}$$

and

$$\frac{\partial \zeta_S}{\partial t} = 0. \tag{51}$$

According to Courant (1951), differing by a constant, the Poisson equation

$$\nabla^2 a = G(\lambda, \varphi) \tag{52}$$

has its solution expressed as

$$a(\lambda, \varphi) = -\frac{1}{2\pi} \int_0^{2\pi} \int_{-\pi/2}^{\pi/2} \ln\sqrt{2(1-\cos\gamma)}\, G(\lambda', \varphi') \cdot \cos\varphi' d\varphi' d\lambda', \tag{53}$$

Here $\cos\gamma = \sin\varphi \sin\varphi' + \cos\varphi \cos\varphi' \cos(\lambda - \lambda')$. If a is replaced by $\partial\psi_S/\partial t$, then from Eq. (51) $\partial\psi_S/\partial t$ will vanish everywhere over the global surface. Furthermore, by mathematical induction, if the 1st to $(n-1)$th derivatives of ψ_S with respect to t vanish everywhere over the surface, then the derivatives in the same order of ζ_S with respect to t, will also vanish everywhere over the same surface. Since

$$\frac{\partial^n \zeta_S}{\partial t^n} = -\sum_{m=0}^{n-1} \frac{(n-1)!}{(n-m-1)!\,m!} \left\{ \frac{\partial^m \boldsymbol{V}_S}{\partial t^m} \cdot \nabla \frac{\partial^{n-m-1} \zeta_S}{\partial t^{n-m-1}} + \frac{\partial^m \boldsymbol{V}_A}{\partial t^m} \cdot \nabla \frac{\partial^{n-m-1} \eta_A}{\partial t^{n-m-1}} \right\} - \mu_2 \frac{\partial^{n-1} \boldsymbol{V}_A}{\partial t^{n-1}} \cdot \nabla z_{0S} + \mu_1 \nabla^2 \frac{\partial^{n-1} \zeta_S}{\partial t^{n-1}} = 0,$$

from the expression (53), we get

$$\frac{\partial^n \psi_S}{\partial t^n} = 0.$$

Hence, if ψ can be expanded by Taylor series in the neighborhood of $t=t_0$, then

$$\psi(\lambda, \varphi, t_0 + \Delta t) = \psi_{0A} + \left(\frac{\partial \psi_A}{\partial t}\right)_0 \Delta t + \cdots + \frac{1}{n!}\left(\frac{\partial^n \psi_A}{\partial t^n}\right)_0 \Delta t^n + \cdots,$$

where the subscripts circles denote the quantities involved at $t=t_0$.

On the other hand, according to the definition of antisymmetry of (3), the 1st to nth derivatives of ψ_A with respect to t are also antisymmetric. Hence, $\psi(\lambda, \varphi, t_0 + \Delta t)$ is antisymmetric. Applying the above results repeatedly and making derivation step by step, we can prove that at any instant $t \geqslant t_0$, ψ is antisymmetric.

Therefore, under condition (48), if the stream function field is antisymmetric at initial instant, then it is impossible to produce any symmetric or asymmetric stream function thereafter.

4. *Asymmetric Motion Caused by Orography*

However, if $\nabla z_{0A} \neq 0$ and condition (49) is valid at initial instant t_0, then Eq. (47) becomes

$$\frac{\partial \zeta_S}{\partial t} = -\mu_2 \boldsymbol{V}_S \cdot \nabla z_{0A}. \tag{54}$$

Evidently, the right-hand side of the above expressions is symmetric. If it is not identically equal to zero everywhere over the global surface, then from expression (53) $\partial\zeta_S/\partial t$ and $\partial\psi_S/\partial t$ would not vanish in general. Therefore, the antisymmetric distribution of orography could cause the generation of ψ_S and the original antisymmetric stream function field would be changed into an asymmetric stream function field.

5. *Asymmetric Motion Caused by Horizontal Diffusion*

(1) $\mu_1=$constant

Under conditions (48) and (49), from the above argument, the 1st to *n*th derivatives of ψ_S with respect to t vanish everywhere over the global surface. Hence, at any instant $t\geqslant t_0$, there is no possibility to produce asymmetric motion.

(2) μ_1 is a function of space

Assume

$$\mu_1=\mu_{1S}+\mu_{1A}, \tag{55}$$

with the similar technique stated previously, Eq. (45) can be separated into

$$\frac{\partial\zeta_A}{\partial t}+\boldsymbol{V}_S\cdot\nabla\eta_A+\boldsymbol{V}_A\cdot\nabla\zeta_S+\mu_2(\boldsymbol{V}_S\cdot\nabla z_{0S}+\boldsymbol{V}_A\cdot\nabla Z_{0A})=\mu_{1A}\nabla^2\zeta_S+\mu_{1S}\nabla^2\zeta_A, \tag{56}$$

and

$$\frac{\partial\zeta_S}{\partial t}+\boldsymbol{V}_S\cdot\nabla\zeta_S+\boldsymbol{V}_A\cdot\nabla\eta_A+\mu_2(\boldsymbol{V}_S\cdot\nabla z_{0A}+\boldsymbol{V}_A\cdot\nabla z_{0S})=\mu_{1S}\nabla^2\zeta_S+\mu_{1A}\nabla^2\zeta_A. \tag{57}$$

If conditions (48) and (49) are still used, then

$$\frac{\partial\zeta_S}{\partial t}=\mu_{1A}\nabla^2\zeta_A. \tag{58}$$

Obviousely, it is possible to produce nonzero ψ_S.

Therefore, for any stream function field which is antisymmetric at initial instant, ψ_S can be produced through orography and diffusion, and then asymmetric motion occurs.

VIII. COMPARISON BETWEEN GLOBAL AND HEMISPHERICAL FORECASTS MADE WITH THE SPECTRAL MODEL

When the forecasts are performed with the spectral model, it is necessary to express the global data used in spectral expansion. However, if there are only hemispheric data, then certain assumptions should be made in order to extend the data to the entire global surface, otherwise we can not expand the data in spectral form. For example, we may assume the predictands to be symmetric or antisymmetric with respect to the equator. In doing so, one may often ask what differences there are between the global and hemispheric forecasts. Although many numerical experiments were made to answer the question, the corresponding theoretical study is lacking. Further, even if the differences exist between such forecasts, it is still unclear what mechanism may cause the differences and what impact on the forecasts the mentioned assumptions may make.

Usually, the assumption that ψ is antisymmetric is employed for the forecasts made with the barotropic filtered model. In this way ζ and v become antisymmetric and u symmetric. On the basis of the above arguments, the comparison between the global and hemispheric forecasts could be theoretically made in the following paragraphs.

Evidently, the above assumption on the hemispheric forecasts is essential to assume the equator to be equivalent to a rigid wall, thus $v=0$. Then the total absolute vorticity, enstrophy and kinetic energy are conservative over the hemispheric domain. In addition, in the case of ignoring orography and horizontal diffusion, the invariance of antisymmetry of ψ also holds for the hemispheric forecasts. All these results are in agreement with those of the global forecasts.

On the other hand, however, there still exist some disagreements between the two kinds of forecasts. They are mainly as follows.

(1) The distributions of ψ and z_0 may be distorted when the assumptions such as ψ being antisymmetric and z_0 symmetric, are made. In some regions of high-latitudes, the distortion of z_0 may be quite serious.

(2) In the case considering orography and horizontal diffusion, assuming z_0 and μ_1 to be symmetric in essence implies the elimination of the mechanism of generation of asymmetric motion and thus no air current crossing the equator would occur.

(3) The hemispheric forecasts can not take account of the interaction between the symmetric and antisymmetric motions. For example, from Eq. (5) the contribution of $-\mathbf{V}_A \cdot \nabla\zeta_S$ to $\partial\zeta_A/\partial t$ disappears. From Eqs. (16) and (25) the conversion between the symmetric and antisymmetric enstrophy, and that between the symmetric and antisymmetric kinetic energy can not happen.

(4) Owing to the limitation of the assumed rigid wall at the equator in the hemispheric forecast, there is no exchange of momentum, energy and vorticity, and interaction of motion between the Northern and Southern Hemispheres.

Of course, the deficiencies mentioned above result from the assumption imposed on the hemispheric forecasts. Therefore, it should be noticed that the quality of forecasts and the simulation of weather evolution will be affected.

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