# PRELIMINARY STUDY OF BAROTROPIC STOCHASTIC DIFFERENCE MODEL APPLIED TO WEATHER PREDICTION

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Received August 23, 1989

ABSTRACT

In order to consider both the deterministic and the stochastic property of atmospheric motion simultaneously, in this paper, the weather prediction is proposed as the problem of the evolution of meteorological field. The historical viewpoint of atmospheric motion is emphasized here. Based on time series analysis technique, a stochastic-dynamical model with multiple initial fields is derived. Thus, weather forecasting is summed up as a problem of solving a set of stochastic difference equations. For the barotropic atmosphere, the numerical solutions of the equations are obtained by using the method of empirical orthogonal functions (EOF), and examples of medium-range weather prediction are given here. Meanwhile, selecting the order of time series, i.e., determining the number of initial fields properly, is also discussed.

**Key words:** barotropic stochastic difference model, empirical orthogonal function, weather forecasting, verification of prediction

## I. INTRODUCTION

Weather prediction infers the future physical state of atmosphere. Obviously, it is a physical problem. But when observing the atmospheric state, people obtain numerous data. The collection, transfer and processing of the data are a typical problem of information science too. In order to predict the future weather accurately, not only the rules of atmospheric motion must be grasped, but also the information in the observed data available has to be picked up as far as possible. In the sense it is called the dual nature of weather prediction. Therefore, there are two kinds of current objective weather prediction techniques. One looks upon weather variation as a fluid system which follows hydrodynamical and thermodynamical law. That is now so-called numerical weather prediction. The other is based on empirical or probabilistic and statistical prediction formulas by utilizing the relationship between present state observed and future state of the atmosphere. And it has a general designation of objective empirical method.

More and more studies indicate that fluid behavior not only has deterministic nature on the one hand, but also has stochastic nature on the other hand. Under the different external conditions the same fluid has different behavior which is deterministic or stochastic or

sometimes which interweaves the former with the latter. As a result, only attention to the two natures of atmospheric motion has been paid simultaneously, can we prepare the best prediction based on available observed data.

Gu (1958) has firstly proposed a special numerical weather prediction utilizing historical data. Chou (1961) has suggested a realizable scheme by solving the generalized solution of differential equations through constituting the historical data set. Zheng and Du (1973) consequently have designed a multiple initial field model and an example was computed. Soon afterwards Guo et al. (1980) proved that continuous evolution of atmospheric temperature and pressure fields is equivalent to the thermal condition of the underlying surface, then a long-range forecasting scheme by utilizing several average circulation maps has been suggested and computed. From the time series viewpoint, Cao (1981) originated an idea to treat statistic-dynamic weather prediction as a problem of solving stochastic difference equations. Investigating the analytic solution of evolution equations and kdV equations, Laiktman (1983) indicates that utilizing several atmospheric states before initial field may compensate the error caused by the deficiency of initial field data and increase atmospheric predictability. These demonstrations and models above mentioned show the necessity and possibility of preparing weather prediction by using several initial fields. This paper is to follow Cao's work to show how to calculate barotropic stochastic-difference model. And, for instance, two 500 hPa medium-range weather predictions are also given.

## II. THE PRINCIPLE OF THE MODEL

Atmospheric motion may be considered as the evolution of continuous medium extending on a limited space domain. As we measure it, the discrete observations, which cover the space-domain with certain time interval, are obtained. We do use these data to show the variation of continuous medium. Therefore, the problem of weather prediction is how to utilize the information included in these data and the physical law of atmospheric motion to infer its future. As a general rule, the differential equation describing continuous field may be written as

$$\frac{\partial x(t_0)}{\partial t} = F(x, \lambda, t_0), \tag{1}$$

where $x$ is a certain physical quantity, $t_0$ is the starting prediction time, $\lambda$ is the parameters of dynamics and/or thermodynamics, and $F(x, \lambda, t)$ showing the space term at $t_0$ is sometimes called the function of disturbance.

The method of time series analysis only utilizes the left side of Eq. (1), i.e., the information of the local physical quantity variation with time. The method of numerical prediction makes the best of the conversion relationship between the space term and the time term in (1), but only the information of one space field is used. If we consider the evolution of the field with discrete observations as a multidimensional time series and utilize rich information involved in a series of space fields, then the problem of weather forecasting is formulated as an evolutional problem of weather process, and the stochastic-dynamical model with a set of initial fields is derived.

Eq. (1) is rewritten in the form of forward difference as

$$\frac{x(t_0+\Delta t) - x(t_0)}{\Delta t} = F(x, \lambda, t_0) \quad . \tag{2}$$

After statistically discreting (2), we obtain

$$x(t_0+1)=\beta_0 x(t_0)+\theta_0 F(x,\lambda,t_0)+\eta_0, \tag{3}$$

where $\eta_0$ is random noise. Similarly we have

$$x(t_0)=\beta_1 x(t_0-1)+\alpha_1 F(x,\lambda,t_0-1)+\eta_1, \tag{4}$$

$$x(t_0-1)=\beta_2 x(t_0-2)+\alpha_2 F(x,\lambda,t_0-2)+\eta_2, \tag{5}$$

$$\cdots\cdots\cdots\cdots\cdots\cdots$$

$$x(t_0-p+1)=\beta_p x(t_0-p)+\alpha_p F(x,\lambda,t_0-p)+\eta_p. \tag{6}$$

Denoting

$$x(t_0-p)\cong F(x,\lambda,t_0-(p+1)), \tag{7}$$

we substitute (4), (5) and (6) into (3) one by one, rewrite the coefficient as $\theta_i (i=0,1,\cdots,p,p+1)$, and combine all noise terms into notation $\eta$. Then we obtain the stochastic difference equation

$$x(t_0+1)=\theta_0 F(x,\lambda,t_0)+\theta_1 F(x,\lambda,t_0-1)+\cdots+\theta_p F(x,\lambda,t_0-p)$$
$$+\theta_{p+1}F(x,\lambda,t_0-(p+1))+\eta=\sum_{i=0}^{p+1}\theta_i F(x,\lambda,t_0-i)+\eta. \tag{8}$$

where

$$\theta_1=\beta_0\alpha_1,\ \theta_2=\beta_0\beta_1\alpha_2,\cdots,\ \theta_p=\beta_0\beta_1\cdots\beta_{p-1}\alpha_p,\ \theta_{p+1}=\beta_0\beta_1\cdots\beta_p.$$

Eq. (8) expresses the prediction for $x$ at $t+1$ by using $p+1$ initial fields, $\theta_0,\theta_1,\cdots,\theta_p$ are the weights of multiple initial fields for predicting $x(t_0+1)$ respectively and are estimated by historical data.

If we use only one initial field, i.e. $p=0$, $\theta_1=\theta_2=\ldots=\theta_p=0$, and let $\theta_0=\Delta t$, $\theta_{p+1}=1$, $\eta=0$. Eq. (8) becomes the deterministic difference equation, i.e., Eq. (2).

Eq. (8) is derived from partial differential equation, so that the model of time series shown by Eq. (8) is different from the others derived from ordinary differential equation, such as autoregressive model, autoregressive moving average model etc. (Box and Jinkins, 1971). The left-hand side of Eq. (8) is the physical quantity at a certain local point and the right-hand side is the initial fields at multiple times. Eq. (8) reveals a full effect of the fields at multiple times on the local point. In order to simplify the calculation, $\eta$ is taken as white noise.

## III. THE BAROTROPIC STOCHASTIC DIFFERENCE MODEL

Let us write the equivalent barotropic vorticity equation at the mth grid point ($m=1,2,\cdots,M$) on an isobaric surface as follows:

$$(\nabla^2-\lambda^2)\frac{\partial z_m}{\partial t}=J\left(\frac{1}{f_m}\nabla^2 z_m,\ z_m\right)-\beta_m\frac{\partial z_m}{\partial t}, \tag{9}$$

where $z_m$ is the geopotential height, $\nabla^2$ is the horizontal Laplacian, $J$ is Jacobian, and $\lambda^2$ is the coefficient controlling the back moving of ultralong wave. Both $f_m=2\sin\varphi_m$ and $\beta_m=2R^{-1}\cos\varphi_m$ are conventional notations in meteorology.

Denote

$$A_m(t_0-t)\equiv[(\nabla^2-\lambda^2)z_m]_{t_0-t}, \tag{10}$$

$$E_{m,t_0-t}\equiv\left[J\left(\frac{1}{f_m}\nabla^2 z_m,\ z_m\right)-\beta_m\frac{\partial z_m}{\partial x}\right]_{t_0-t},$$

where $t_0$ is the initial time, and $t=-1,0,\cdots,p$. Then Eq. (8) becomes

$$A_m(t_0+1)=\theta_{0,m}E_{m,t_0}+\theta_{1,m}E_{m,t_0-1}+\cdots+\theta_{p,m}E_{m,t_0-p}$$
$$+\theta_{p+1,m}E_{m,t_0-(p+1)}+\eta. \tag{11}$$

Evidently, the stochastic difference equation with $M$ dimensions will be obtained for $M$ grids as

$$A(t_0+1)=\sum_{t=0}^{p+1}\theta_t E_{t_0-t}+\eta \ , \tag{12}$$

where $\theta_t$ is a weight coefficient matrix with $M\times M$ orders, $A(t_0+1)$ and $E_{t_0-t}$ are vector with $M\times 1$ orders, and $E_{t_0-(p+1)}\equiv A(t-p)$. Thus the contour at the time $(t_0+1)$ can be got from the contours at $t_0,\cdots,t_0-p$ by solving Eq. (12).

## IV. THE METHOD SOLVING STOCHASTIC DIFFERENCE EQUATION

In general, there are $10^2$—$10^3$ grid points in a field, so that Eq. (12) is a time series with very high dimension. It is very difficult to solve the equation directly. For simplification of the computation, it is assumed that $\theta$ is independent of grid point but time. Thus Eq. (12) becomes the time series with one dimension. Further the EOF is used to decompose the height field so as to make the problem become the prediction for principal components.

### 1. *Decomposing the Height Field with EOF*

The height $z_{m,t_0-t}$ ($t=-1, 0,\cdots,p$, and $m=1, 2,\cdots, M$) can be decomposed as follows

$$z_{m,t_0-t}=\sum_{\nu=1}^{S}b_{m,\nu}C_{\nu,t_0-t}, \qquad t=-1,0,\cdots,p \tag{13}$$

where $b_{m,\nu}$ is EOF , i.e. the space function, $C_{\nu,t_0-t}$ is the principal component, i.e. the time-dependent coefficient, and $s$ is the truncation point of the EOF series. Substituting (13) into (11) and (12), and putting it in order, we get

$$\sum_{\nu=1}^{S}(\nabla^2 b_{m,\nu}-\lambda^2 b_{m,\nu})C_{\nu,t_0+1}=\sum_{t=0}^{p+1}\theta_t\tilde{E}_{m,t_0-t}+\eta, \tag{14}$$

where

$$\tilde{E}_{m,t_0-t}=\sum_{\nu=1}^{S}\sum_{\mu=1}^{S}J\left(\frac{1}{f_m}\nabla^2 b_{m,\nu},b_{m,\mu}\right)C_{\nu,t_0-t}C_{\mu,t_0-t}$$

$$-\beta_m\sum_{\nu=1}^{S}\frac{\partial b_{m,\nu}}{\partial x}C_{\nu,t_0-t}\ . \qquad m=1,2,\cdots,M \tag{15}$$

### 2. *Estimation of the Coefficient*

We integrate (15) for the whole space $\sigma$ and let

$$g_\nu=\int_\sigma(\nabla^2 b_{m,\nu}-\lambda^2 b_{m,\nu})d\sigma \ , \tag{16}$$

$$l_{t_0-t}=\int_\sigma\tilde{E}_{m,t_0-t}d\sigma \ , \tag{17}$$

then we obtain

$$\sum_{\nu=1}^{S}g_\nu C_{\nu,t_0+1}=\sum_{t=0}^{p+1}\theta_t l_{t_0-t}+\eta \ . \tag{18}$$

$N(\gg p+1)$ height fields are taken to determine $\theta_t$. Let

$$\varepsilon^2=\frac{1}{N-p-1}\sum_{t_0=p+1}^{N-1}\left[\sum_{\nu=1}^{S}g_\nu C_{\nu,t_0+1}-\sum_{t=0}^{p+1}\theta_t l_{t_0-t}\right]^2 . \tag{19}$$

From $\partial\varepsilon^2/\partial\theta_t=0$, $t=0,1,\cdots,p+1$, a set of normal equations of $\theta_t$ is obtained as

$$\sum_{t=0}^{p+1} l_{qt}\hat{\theta}_t = u_q \quad , \tag{20}$$

where

$$l_{qt} = \frac{1}{N-p-1}\sum_{t_0=p+1}^{N-1} l_{t_0-q}\, l_{t_0-t} \quad , \tag{21}$$

$$u_q = \frac{1}{N-p-1}\sum_{t_0=p+1}^{N-1}\sum_{\nu=1}^{S} g_\nu C_{\nu,t_0+1}\, l_{t_0-q} \quad . \tag{22}$$

$$(q=0,1,\cdots,p+1)$$

3. *The One-Step Prediction of Principal Component*

Using $\hat{\theta}_t$ obtained from historical data, we sum up (14) for $M$ grid points and let

$$e^2 = \frac{1}{M}\sum_{m=1}^{M}\left[\sum_{\nu=1}^{S}(\nabla^2 b_{m,\nu} - \lambda^2 b_{m,\nu})\, C_{\nu,t_0+1} - \sum_{t=0}^{p+1}\hat{\theta}_t \tilde{E}_{m,t_0-t}\right]^2 . \tag{23}$$

From $\partial e^2/\partial C_{\nu,t_0+1}=0$, $\nu=1,2,\cdots,S$, the set of normal equations to estimate $C_{\nu,t_0+1}$ is obtained as

$$\sum_{\nu=1}^{S} \gamma_{\mu,\nu}\hat{C}_{\nu,t_0+1} = d_\mu, \mu,\nu=1,2,\cdots,S, \tag{24}$$

where

$$\gamma_{\mu,\nu} = \frac{1}{M}\sum_{m=1}^{M}(\nabla^2 b_{m,\mu} - \lambda^2 b_{m,\mu})(\nabla^2 b_{m,\nu} - \lambda^2 b_{m,\nu}) \quad , \tag{25}$$

$$d_\mu = \frac{1}{M}\sum_{m=1}^{M}(\nabla^2 b_{m,\mu} - \lambda^2 b_{m,\mu})\sum_{t=0}^{p+1}\theta \tilde{E}_{t_0-t} \quad . \tag{26}$$

When we substitute the $C_{\nu,t_0+1}$ solved into (13), the one step prediction of height field may be made.

Obviously, when taking $p=0$, $\theta_0=\Delta t$, $\theta_1=1$, and $\eta=0$ in Eq.(11), Eq. (24), is equivalent to Sellers (1957) statistic-dynamic prediction equation.

## V. DETERMINATION OF EOF FROM A LARGE SAMPLE

This model is used to forecast the Northern Hemisphere contour, so that the EOFs of 540 longitude-latitude mash grid points between 10°N and 80°N must be determined. And $\theta_t$ is estimated by using the data about 150 days. As a result, both time-domain and space-domain are so big that we need a computer with very big memory space to solve the EOF directly. For this reason, the method proposed by Rinne and Karhila (1979) is employed. Here we briefly describe how to use the method in this paper.

If the grid is divided into $2^k$ subgrids, $k=2$ in this paper, it is divided into four subgrids. The EOF of each subgrid can be easily calculated.

Then one obtains

$$z^i(x,t) = z_m^i(x) + \sum_{\nu=1}^{N^i} b_\nu^i(x)\, C_\nu^i(t) \quad , \tag{27}$$

$$z^j(x,t) = z_m^j(x) + \sum_{\nu=1}^{N^j} b_\nu^j(x)\, C_\nu^j(t) \quad , \tag{28}$$

where $i$ and $j$ express the $i$th and $j$th subgrid respectively, $z^i(x, t)$ and $z^j(x, t)$ represent the geopotential height of corresponding subgrids, $z_m$ is the height of mean field computed from continuous historical data required, $b_\nu^i(x)$ and $b_\nu^j(x)$ are EOFs, $C_\nu^i(t)$ and $C_\nu^j(t)$ are principal components, and $N^i$, $N^j$ are the truncation points of the EOF-series.

The geopotential height $z^{ij}$ on the combined grid area $ij$ can be approximately expressed by

$$z^{ij}(x,t) \approx z_m^{ij}(x) + \sum_{\nu=1}^{N^i} b_\nu^i(x) C_\nu^i(t) + \sum_{\nu=1}^{N^j} b_\nu^j(x) C_\nu^j(t) \ , \tag{29}$$

where assuming $b_\nu^i(x)=0$ if $x \in j$; $b_\nu^j(x)=0$ if $x \in i$.

Owing to strong correlation of $C_\nu^i(t)$ and $C_\nu^j(t)$, there is still redundant information in Eq.(29). The coefficients $C_\nu^i$ and $C_\nu^j$ are orthogonalized by a new set of orthogonal functions. If both $N^i$ and $N^j$ are not over half of the number of subgrid points, the metrix formed by computing the covariances between coefficient time series $C_\nu^i$ and $C_\nu^j$ will not exceed the maximum memory space. Therefore, we can solve the eigenvectors $J_\nu(\mu)$ of the covariance of the matrix $[C_\nu^i(t), C_\nu^j(t)]^T$, then obtain the principal component:

$$C_\mu^{ij}(t) = \sum_{\nu=1}^{N^i} J_\nu(\mu) C_\nu^i(t) + \sum_{\nu=1}^{N^j} J_{N^i+\nu}(\mu) C_\nu^j(t) \ , \tag{30}$$

where $J_\nu(\mu)$ is called joining function. In the same way, we can also obtain the new EOF on the combined grid area $ij$.

$$b_\mu^{ij}(x) = \sum_{\nu=1}^{N^i} J_\nu(\mu) b_\nu^i(x) + \sum_{\nu=1}^{N^j} J_{N^i+\nu}(\mu) b_\nu^j(x) \ , \tag{31}$$

where $x \in ij$; $b_\nu^i(x)=0$, if $x \in j$; $b_\nu^j(x)=0$, if $x \in i$. It is easy to prove that

$$z^{ij}(x,t) \approx z_m^{ij}(x) + \sum_{\mu=1}^{N^{ij}} b_\mu^{ij}(x) C_\mu^{ij}(t) \ , \tag{32}$$

where $N^{ij}$ is the truncation point of the combined grid area $ij$. The above calculation is repeated $2^k-1$ times, and the whole field can be decomposed. If in each step we take the first half of all components, the elements of covariance matrix equal $1/2^k$ of the number of grid points, so that determining EOF from a large sample does not need a computer with huge memory space any more. Since the EOF series converges very fast, even though the second half of all components is omitted, in this paper only a fraction of 2—3% of the original variance is lost and this fraction is mainly of noise and error.

## VI. DETERMINING THE ORDER OF TIME SERIES

To determine the order $p+1$ of time series, in other words, how many pieces of continuous synoptic maps are needed to make the superior prediction? It is a critical problem in the prediction method of multiple initial fields. We choose statistically the order based on the principle of minimum prediction error.

Let

$$er(p+1) = \frac{1}{(N-p-1)M} \sum_{t_0=p+1}^{N-1} \sum_{m=1}^{M} (z_{m,t_0+1} - \hat{z}_{m,t_0+1}^{(p+1)})^2 \ , \tag{33}$$

where $z_{m,t_0+1}$ is an observation, and $\hat{z}_{m,t_0+1}^{(p+1)}$ is a forecast. Having solved $\hat{\theta}_i$ and $C_{\nu,t_0+1}$, one can calculate $er$ for numerous orders $p$, and certainly find the statistical

optimal order $p+1$. But this procedure needs too much work to do. And the order which is obtained in this way is determined in sense of averaging the historical sample. In order to make the obtained order of time series have distinct physical meaning, in a case study we determine it from synoptic and dynamic analysis. Here two examples are given as follows.

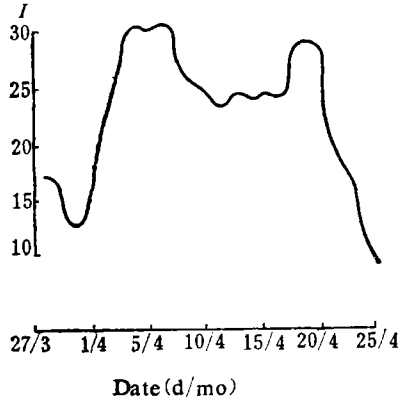


Fig. 1. Zonal index I (unit: 10gpm) along 55°N from 0°E to 140°E, 27 March to 25 April 1974.

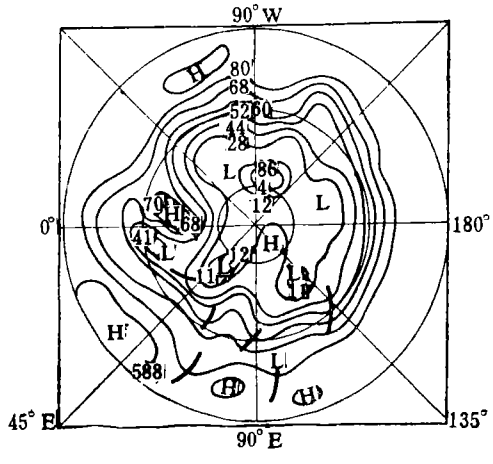


Fig. 3. Observed 500 hPa height map at 1200 GMT 18 April 1974, unit: 10 gpm.

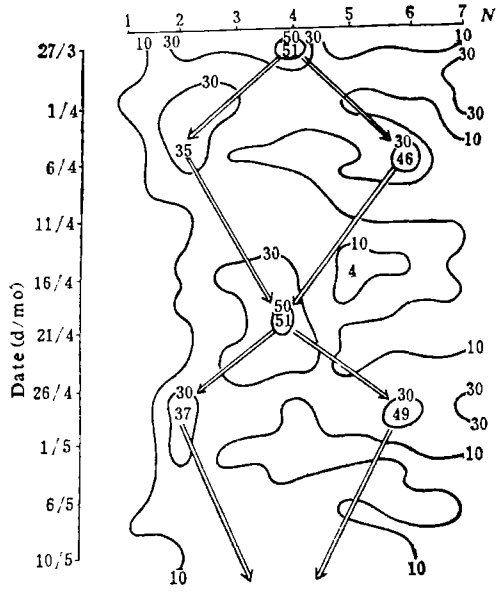


Fig. 2. Time cross-section of total eddy kinetic energy spectrum from 27 March to 10 May 1974, N denotes wavenumber.

## 1. *The Developing Process of Large-Scale Disturbance*

The zonal circulation pattern over the Northern Hemisphere transformed into the meridional circulation from 19 to 23 April 1974.After a persistent high value lasted in the first decade of April, the index of zonal circulation along 55°N in Eurasia(0°—140°E) became descending tendency notably (Fig.1). On the 500 hPa level over the Northern Hemisphere the European trough developed deeply and the Ural ridge moved eastward and the trough of East Asia became sharper and sharper (Figs. 3—5a).The zonal current downstream from the Atlantic blocking high decreased, and the meridional circulation prevailed in Eurasia instead. We have analyzed the mean total eddy kinetic energy spectrum of 65°N, 55°N and 45°N in the period of 27 March to 10 May (Fig. 2) and found that there was a harmonic oscillation period

in the range of the ultralong wave and long wave range from 27 March to 18 April. On 27 March the main centre of 510m²/s² appeared on the 4th wave. The strength of the centre decreased as time went on and the centre split and transferred to the 2nd and the 6th wave. Then the subcentres of 350 and 460m²/s² appeared on both the 2nd wave and the 6th wave, respectively. Later on, the energy of the 4th wave was excited again and gained energy from other waves. On 18 April a main centre of 510 m²/s² appeared again on the 4th wave. The resonance phenomenon of eddy energy matches the time scale of medium-range weather process. According to this analysis, the spell during which the energy was accumulated, is nearly equal to half of the period of energy harmonic oscillation and is taken as the order $p+1$ of time series, i.e. fifteen initial field maps from 4 April to 18 April is taken.

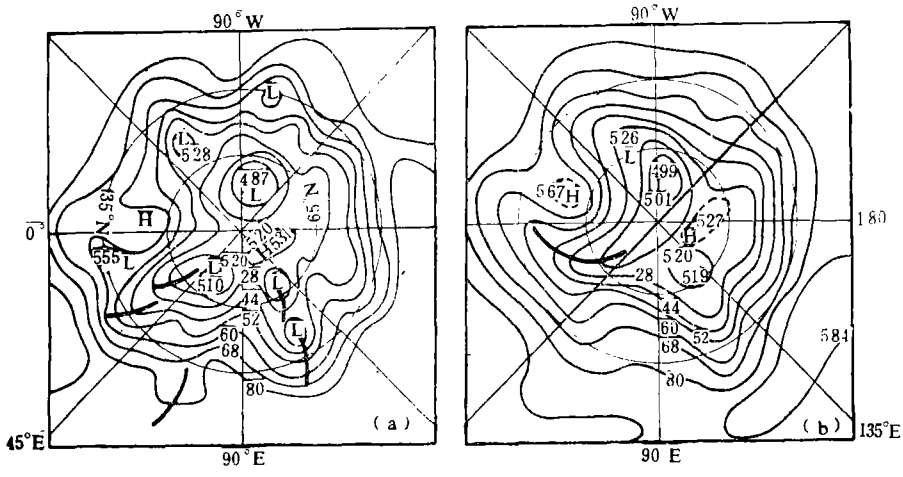


Fig. 4. Observed (a) and prediction (b) 500 hPa map (unit: 10 gpm) at 1200 GMT 20 April 1974.

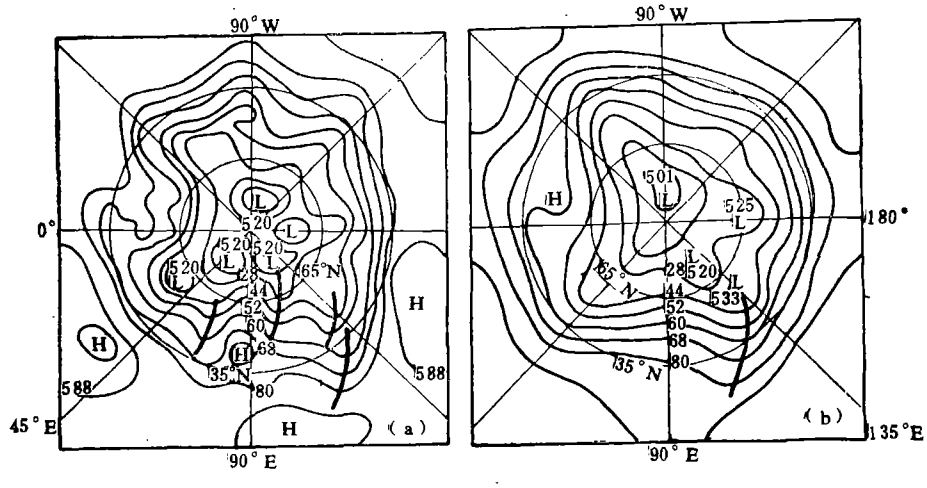


Fig. 5. As in Fig. 4 except for 1200 GMT 22 April 1974.

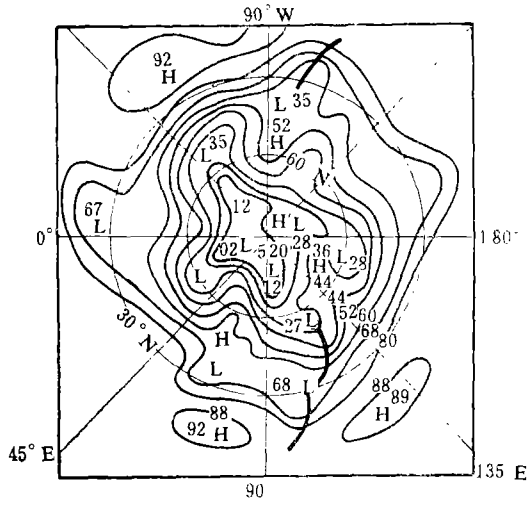


Fig. 6. Observed 500 hPa height map at 1200 GMT 13 April 1975.

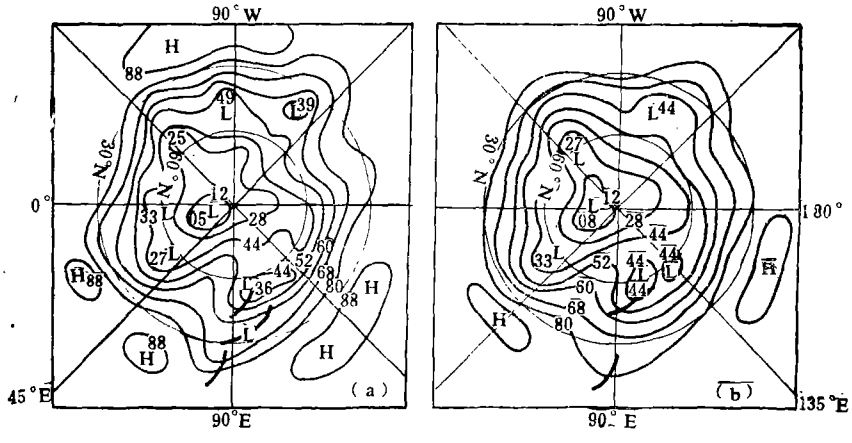


Fig. 7. As in Fig. 3 except for 1200 GMT 15 April 1975.

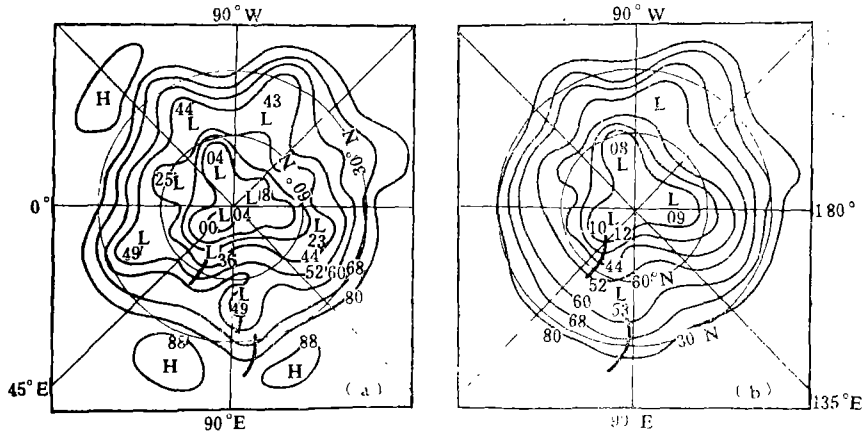


Fig. 8. As in Fig. 3 except for 1200 GMT 17 April 1975.

2. *The Damping Process of Large-Scale Disturbance*

Another example is taken to study the stochastic difference model. On the contrary of the above example, from 13 (Fig. 6) to 17 April (Fig. 8a) 1975, the meridional circulation over the Northern Hemisphere changed to zonal current. Through a low index period in the middle decade of April, the index began to get ascending tendency. Meanwhile, on the 500 hPa map of the Northern Hemisphere the flow pattern with four waves (Fig. 6) gradually turned into the flow pattern with three waves (Fig. 8a). Similar to the above case, the half of the energy harmonic oscillation period is taken the order $p+1$ of time series, i.e. eleven initial field maps from 3 to 13 April.

3. *Calculation Procedure*

(1) The daily data of the geopotential height at $5° \times 5°$ longitude-latitude grid on the 500 hPa level over the Northern Hemisphere are used as data base. There are 540 grid points in all between 10°N—80°N.

(2) The boundary condition is taken as follows: The Arctic height is equal to the mean of the geopotential heights along latitude 80°N and the heights along latitude 5°N is the same as 10°N.

(3) The EOFs of subsequent 150-day data before 18 April 1974 and 13 April 1975 are respectively calculated by using the method described in Section V. Then 16 and 12 estimations of $\hat{\theta}_t (t=0,1,\cdots,15$ and $0,1,\cdots,11)$ are obtained respectively.

(4) After solving $C_{\nu,t_0+1}$ and $z_{m,t_0+1}$ $(m=1\ 2,\cdots,540)$, we used the prediction as initial field to do extended prediction repeatedly. Finally we obtain the prediction for 24, 48, 72, 96 h projection respectively.

## VII. VERIFICATION

The verification for prediction maps has been made by using the 1200 GMT 500 hPa map from 20 to 22 April 1974 and from 15 to 17 April 1975 respectively. The observed and the predicted 500 hPa for 48h and 96 h projection are given in Figs. 4, 5 and 7, 8 for two examples. We can find evidently that the predictions of circulation within 48 h are in good agreement with the observation, the forecast of the main centre of high and low approaches the observed approximately. Although the prediction circulation for 96 h is smoother than real one, the main topography in the extended range is still similar to the observations. Especially during the four days of the former example the increase of subtropical high over the whole Northern Hemisphere are also predicted correctly. Further quantitatively checking indicates that the averaged root mean square error and anomaly correlation of prediction circulation with real observed one accordingly are 62 gpm and 0.8 for 48h, 87 gpm and 0.75 for 96h, respectively. As a preliminary study of barotropic stochastic difference model, the results are satisfactory. They also prove that this model is capable of making medium-range prediction of large-scale disturbance which is either of developing process or of damping process.

Although it is hard to say that either deterministic or stochastic property attaches major importance to short-medium range weather prediction, in this paper weather developing physics proccess is revealed in the viewpoint of energy harmonic fluctuation and its information is then input by using corresponding historical data, so that the satisfactory results of the experiments are enough to prove the correctness and effectiveness of the method of combining historical data and dynamic equations.

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