EFFECTS OF OROGRAPHY AND HORIZONTAL DIFFUSION ON THE GENERATION OF THE ASYMMETRIC MOTION IN THE BAROTROPIC FILTERED MODEL ATMOSPHERE

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ABSTRACT

By utilizing the barotropic vorticity equation including effects of orography and horizontal diffusion, the linearized equations describing symmetric and antisymmetric motions and their analytic solutions are presented. It can be found from the solutions that no matter what kind of motion may be, each solution consists of three waves, namely, Rossby wave related to initial values, marching wave propagating at Rossby wave velocity and stationary wave. The latter two are closely related to orography and horizontal diffusion. However, if the motion is symmetric at the initial instant, then the antisymmetric components of orography and of horizontal diffusion are likely to lead to the generation of antisymmetric motion. In the steady state, the symmetric flow is connected with symmetric orography and horizontal diffusion and the antisymmetric flow with antisymmetric orography and horizontal diffusion. Further, in order to verify the above analysis, three numerical experiments have been made. The results show that antisymmetric orography can produce antisymmetric motion. Finally, the atmospheric interactions between Northern and Southern Hemispheres are discussed.

I. INTRODUCTION

By using the results of analyzing the climatic data in the former article (Liao, 1985), it was pointed out that the symmetric motion is the dominant motion in the atmosphere, while the antisymmetric motion is the secondary one. Nevertheless, the latter is not steady, but varies with time and space. In some abnormal situations, such as the position of the subtropical high ridge or that of ITCZ, deviating from the normal position quite obviously, the antisymmetric motion often changes abnormally. Therefore, studying the mechanism of generation of the antisymmetric motion is of vital significance for the atmospheric general circulation, the formation of climate or long-range weather forecasting.

In another article (Liao and Zou, 1986), when discussing the mechanism of generation of the antisymmetric motion, we deduced that the antisymmetric distribution of orography and of the coefficient of horizontal diffusion is likely the cause to produce the motion. This article is a continuation of that one. It aims at verifying the validity of such deduction by analytic solutions and numerical experiments.

For the above purpose, we shall study the deduction thereinafter by adopting the barotropic vorticity equation including effects of orography and horizontal diffusion. In the second section, adopting the technique given by the author (1985), we shall separate the equation into two linearized equations to describe the symmetric and antisymmetric motions, respectively. In the third and the fourth section the analytic solutions corresponding to

the symmetric and antisymmetric motions are given and analyzed. Finally, the generation of the cross-equatorial flow is discussed.

II. EQUATIONS DESCRIBING THE SYMMETRIC AND ANTISYMMETRIC MOTIONS

The barotropic vorticity equation including effects of orography and horizontal diffusion may be written in the form

$$\frac{\partial}{\partial t} \nabla^2 \psi + V \cdot \nabla (\nabla^2 \psi + f) + \nu_1 V \cdot \nabla z_0 = \nu_2 \nabla^4 \psi, \qquad (1)$$

where ψ stands for the stream function; V the horizontal wind; f the Coriolis parameter; z_0 the surface height above sea level; $v_1 = \hat{a}f/H$, $H = RT_0/g$, being assumed as a constant; $\hat{a} = |V_0|/|V|$; v_2 the coefficient of horizontal diffusion. Note that the Coriolis parameter f involved in v_1 is a function of latitude and has not been assumed as a constant as in usual treatment. This is because the constant assumption is unreasonable for a global problem.

Similar to Liao and Zou (1985, 1986), any scalar quantity $F(\lambda, \varphi, t)$ can be expressed by

$$F = F_S + F_A \tag{2}$$

where

$$F_{S}(\lambda, \varphi, t) = F_{S}(\lambda, -\varphi, t), \qquad (3)$$

and

$$F_{A}(\lambda, \varphi, t) = -F_{A}(\lambda, -\varphi, t), \qquad (4)$$

where λ stands for the longitude, φ the latitude; the subscripts S and A stand for the symmetric and antisymmetric components of F, respectively. As for wind V, we have

$$V = V_s + V_A, \tag{5}$$

where

$$\mathbf{V}_{S} = \mathbf{K} \times \nabla \psi_{A} = u_{S} \mathbf{i} + v_{A} \mathbf{j}, \tag{6}$$

and

$$\mathbf{V}_{A} = \mathbf{K} \times \nabla \psi_{S} = u_{A} \mathbf{i} + v_{S} \mathbf{j}. \tag{7}$$

It can be seen from the definitions of V_S and V_A that ψ_A or $\nabla^2 \psi_A$ represents the symmetric flow field, namely, the symmetric motion; ψ_S or $\nabla^2 \psi_S$ the antisymmetric flow field, namely, the antisymmetric motion. Therefore, the symmetry of the stream function is contrary to that of the flow field or of motion.

According to Liao (1985), we assume

$$\bar{u}_S \gg \bar{u}_A,$$
 (8)

and consider the case that

$$\nabla^4 \psi_A \gg \nabla^4 \psi_S, \tag{9}$$

where () stands for the zonal mean. Then replacing F by ψ and z_0 in turn, substituting (2) and (5) into Eq. (1), setting $v_z = b_0 + b_1 \sin \varphi$ and neglecting small terms give the following linearized vorticity equations describing the symmetric and antisymmetric motions.

The symmetric motion

$$\left(\frac{\partial}{\partial t} + \alpha \frac{\partial}{\partial \lambda}\right) \nabla^2 \psi_A + \frac{2(\Omega + \alpha)}{\alpha^2} \frac{\partial \psi_A}{\partial \lambda} + \nu_1 \alpha \frac{\partial z_{0S}}{\partial \lambda} = b_0 \nabla^4 \psi_A, \tag{10}$$

and the antisymmetric motion

$$\left(\frac{\partial}{\partial t} + a \frac{\partial}{\partial \lambda}\right) \nabla^2 \psi_S + \frac{2(\Omega + \alpha)}{a^2} \frac{\partial \psi_S}{\partial \lambda} + \nu_1 \alpha \frac{\partial z_{0A}}{\partial \lambda} = b_0 \nabla^4 \psi_S + b_1 \mu \nabla^4 \psi_A, \tag{11}$$

where Ω stands for the angular velocity of the earth rotation; α the angular velocity of the basic current relative to the earth; $\mu = \sin \varphi$.

III. SOLUTIONS OF THE SYMMETRIC MOTION

 ψ_A and z_{0S} may be expressed by

$$\psi_{A} = a^{2} \sum_{m,n_{1}} \psi_{A}, \, \gamma_{n_{1}}^{m}(t) e^{im\lambda} P_{n_{1}}^{m}(\mu)$$
 (12)

and

$$z_{0.5} = \sum_{m_1, n_2} z_{.5, n_2} e^{im j} P_{n_2}^m(\mu)$$
 (13)

respectively, where
$$P_n^m(\mu)$$
 stands for the normalized associated Legendre polynomial
$$\sum_{m,n_1=1}^M \sum_{m=-M}^{M} \sum_{n_1=1}^{\lfloor m\rfloor+I_1} \sum_{m=1}^{M} \sum_{m=2}^{M} \sum_{m=-M}^{M} \sum_{n_2=\lfloor m\rfloor+\hat{n}_2}^{\lfloor m\rfloor+I_2},$$

where \hat{n}_1 and \hat{n}_2 are odd and even numbers, respectively; $J_1 = J_2 + 1$, J_1 and J_2 are odd and even positive integers, respectively.

Substituting (12) and (13) into Eq. (10) and utilizing the orthogonality of spherical harmonics and the recurrence formula of the associated Legendre polynomial lead to the ordinary differential equation

$$\frac{-d\psi_{A}, \frac{m}{n_{1}}}{dt} + i\sigma_{n_{1}}^{m}\psi_{A}, \frac{m}{n_{1}} = i(\hat{a}_{n_{1}}^{m}z_{S}, \frac{m}{n_{1}+1} + \hat{b}_{n_{1}}^{m}z_{S}, \frac{m}{n_{1}+1}), \tag{11}$$

where

$$\sigma_{\frac{n_1}{n_1}}^m = m \left(\alpha - \frac{2(\Omega + \alpha)}{n_1(n_1 + 1)} - i \frac{b_n n_1(n_1 + 1)}{m a^2} \right),$$

$$\dot{a}_{\frac{n_1}{n_1}}^m = \frac{m \hat{v}_1 \alpha}{n_1(n_1 + 1)} \sqrt{\frac{(n_1 + 1)^2 - m^2}{4(n_1 + 1)^2 - 1}},$$

$$\dot{b}_{\frac{n_1}{n_1}}^m = \frac{m \hat{v}_1 \alpha}{n_1(n_1 + 1)} \sqrt{\frac{n_1^2 - m^2}{4n_1^2 - 1}},$$

$$\hat{v}_1 = \frac{2\Omega \hat{a}}{H} .$$

Solving the above equation we have

$$\psi_{A}, \tilde{\pi}_{1} = \frac{1}{\sigma_{n_{1}}^{m_{1}}} (\hat{a}_{n_{1}}^{m_{1}} z_{N}, \tilde{\pi}_{1+1}^{m_{1}} + \hat{b}_{n_{1}}^{m_{1}} z_{N}, \tilde{\pi}_{1-1}^{m_{1}}) + Ce^{-i\sigma_{n_{1}}^{m_{1}}t}, \tag{15}$$

thus

$$\psi_{\mathcal{A}} = \sum_{m_1 = m_1} \left\{ \frac{1}{\sigma_{n_1}^m} \left(\hat{a}_{n_1}^m z_S, m_{1+1}^m + b_{n_1}^m z_S, m_{1+1}^m \right) + Ce^{-i\sigma_{n_1}^m t} \right\} e^{im\lambda} P_{n_1}^m. \tag{16}$$

Set

$$\psi_{A}(0) = \sum_{m,n} \psi_{A}, m_{n}(0) e^{im\lambda} P_{n}^{m}$$

at the initial instant. Then

$$C = \psi_A, _{n_1}^m(0) - \frac{1}{\sigma_{n_1}^m}(\hat{a}_{n_1}^m z_S, _{n_1+1}^m + \hat{b}_{n_1}^m z_S, _{n_1-1}^m) .$$

Substituting C into (16), we have

$$\psi_{A} = \sum_{m,n_{1}} \left\{ \psi_{A}, {}_{n_{1}}^{m}(0) e^{-i\sigma_{n_{1}}^{m}i} + \frac{1}{\sigma_{n_{1}}^{m}} (\hat{a}_{n_{1}}^{m}z_{S}, {}_{n_{1}+1}^{m} + \hat{b}_{n_{1}}^{m}z_{S}, {}_{n_{1}-1}^{m}) \right. \\ \times \left. \left(1 - e^{-i\sigma_{n_{1}}^{m}i} \right) \right\} e^{im\lambda} P_{n_{1}}^{m}. \tag{17}$$

Now we analyze the physical meaning of the solution (17).

The first term in the braces in the right-hand side of (17) may be written in the form

$$\psi_{A},_{n_{1}}^{m}(0)e^{-i\hat{\sigma}_{n_{1}}^{m}t} = \psi_{A},_{n_{1}}^{m}(0)e^{-i\hat{\sigma}_{n_{1}}^{m}t}e^{-\hat{b}_{0}t}, \qquad (18)$$

where $\hat{b}_0 = b_0 n_1 (n_1 + 1)/a^2$, $\hat{\sigma}_{n_1}^m = m(\alpha - 2(\Omega + \alpha)/n_1(n_1 + 1))$.

The expression demonstrates that the initial disturbance propagates eastward at a speed of $\hat{\sigma}_{n_1}^m/m$, and decays with time by a factor of $e^{-\hat{\delta}_0 t}$. This is consistent with the general property of horizontal diffusion. The term represents the Rossby wave.

The second term in the braces in the right-hand side of (17) may be written in the form

$$\frac{1}{\sigma_{n_{1}}^{m}} (\hat{a}_{n_{1}}^{m} z_{S}, _{n_{1}+1}^{m} + \hat{b}_{n_{1}}^{m} z_{S}, _{n_{1}-1}^{m}) e^{im\lambda} P_{n_{1}}^{m} \\
- \frac{1}{\sigma_{n_{1}}^{m}} (\hat{a}_{n_{1}}^{m} z_{S}, _{n_{1}+1}^{m} + \hat{b}_{n_{1}}^{m} z_{S}, _{n_{1}-1}^{m}) e^{i(m\lambda - \hat{\sigma}_{n_{1}}^{m})} e^{-\hat{b}_{0}t} P_{n_{1}}^{m} .$$
(19)

Evidently, the first term in the above represents the stationary orographical wave; the second term the marching orographical wave propagating eastward at a speed of $\hat{\sigma}_{n_1}^m/m$ and decaying with time.

However, irrespective of stationary or marching orographical wave, they are both symmetric. If the original motion is symmetric, then those orographical waves can not affect the symmetry of the motion.

IV. SOLUTIONS OF THE ANTISYMMETRIC MOTION

Like the case of symmetric motion, the solutions of the antisymmetric motion and the surface height above sea level may be assumed in the forms

$$\psi_{S} = \sum_{m_{1}, n_{2}} \psi_{S}, \, _{n_{2}}^{m}(t) \, e^{i \, m_{\lambda}} P_{n_{2}}^{m}(\mu) \tag{20}$$

and

$$z_{0A} = \sum_{m, n_1} z_{A, n_1}^m e^{i m \lambda} P_{n_1}^m(\mu) . \qquad (21)$$

Substituting (20) and (21) into Eq. (11) we can obtain the ordinary differential equation

$$\frac{d\psi_{S,\frac{m_2}{n_2}}}{dt} + i\hat{\sigma}_{n_2}^{m}\psi_{S}, \quad u_2 = i(\hat{a}_{n_2}^{m}z_A, u_{2+1}^{m} + \hat{b}_{n_2}^{m}z_A, u_{2-1}^{m})
+ (\hat{c}_{n_2}^{m}\psi_A, u_{2+1}^{m} + \hat{d}_{n_2}^{m}\psi_A, u_{2-1}^{m}),$$
(22)

where

$$\begin{split} \dot{a}_{\frac{n}{2}}^{m} &= \frac{m\widehat{v}_{1}\alpha}{n_{2}(n_{2}+1)} \sqrt{\frac{(n_{2}+1)^{2}-m^{2}}{4(n_{2}+1)^{2}-1}} , \\ \hat{b}_{\frac{n}{2}}^{m} &= \frac{m\widehat{v}_{1}\alpha}{n_{2}(n_{2}+1)} \sqrt{\frac{n_{2}^{2}-m^{2}}{4n_{2}^{2}-1}} , \\ \hat{c}_{\frac{n}{2}}^{m} &= m \left(\alpha - \frac{2(\Omega+\alpha)}{n_{2}(n_{2}+1)} - i \frac{b_{0}n_{2}(n_{2}+1)}{m\alpha^{2}} \right) , \\ \hat{c}_{\frac{n}{2}}^{m} &= b_{1} \frac{(n_{2}+1)(n_{2}+2)^{2}}{a^{2}n_{2}} \sqrt{\frac{(n_{2}+1)^{2}-m^{2}}{4(n_{2}+1)^{2}-1}} , \\ d_{\frac{n}{2}}^{m} &= b_{1} \frac{n_{2}(n_{2}-1)^{2}}{a^{2}(n_{2}+1)} \sqrt{\frac{n_{2}^{2}-m^{2}}{4n^{2}-1}} . \end{split}$$

Solving the above equation gives

$$\psi_{S}, \stackrel{m}{}_{n_{2}} = \left\{ \psi_{S}, \stackrel{m}{}_{n_{2}}(0) - \frac{1}{\hat{G}_{n_{2}}^{m}} (\hat{a}_{n_{2}}^{m} z_{A}, \stackrel{m}{}_{n_{2}+1} + \hat{b}_{n_{2}}^{m} z_{A}, \stackrel{m}{}_{n_{2}-1} - i (\hat{c}_{n_{2}}^{m} \psi_{A}, \stackrel{m}{}_{n_{2}+1}) + \hat{d}_{n_{2}}^{m} \psi_{A}, \stackrel{m}{}_{n_{2}-1}) \right\} e^{-i \hat{\sigma}_{n_{2}}^{m} t} + \frac{1}{\hat{\sigma}_{n_{2}}^{m}} (\hat{a}_{n_{2}}^{m} z_{A}, \stackrel{m}{}_{n_{2}+1}) + \hat{b}_{n_{2}}^{m} z_{A}, \stackrel{m}{}_{n_{2}-1} - i (\hat{c}_{n_{2}}^{m} \psi_{A}, \stackrel{m}{}_{n_{2}+1} + \hat{d}_{n_{2}}^{m} \psi_{A}, \stackrel{m}{}_{n_{2}-1})). \tag{23}$$

Thus

$$\psi_{S} = \sum_{m_{s}=n_{2}}^{2} \left\{ \psi_{S}, {}^{m}_{n_{2}}(0) e^{-i\hat{\sigma}^{m}_{n_{2}}t} + \frac{1}{\hat{\sigma}^{m}_{n_{2}}} (\hat{a}^{m}_{n_{2}} z_{\perp, n_{2}+1} + \hat{b}^{m}_{n_{2}} z_{\perp, n_{2}+1} - i(\hat{c}^{m}_{n_{2}} \psi_{\perp, n_{2}+1} + \hat{d}^{m}_{n_{2}} \psi_{\perp, n_{2}-1})) (1 - e^{-i\hat{\sigma}^{m}_{n_{2}}t}) \right\} e^{im\lambda} P^{m}_{n_{2}}.$$
 (21)

It will be seen from the above expression that the antisymmetric motion is connected with the antisymmetric distribution of the surface height above sea level and the coefficient of horizontal diffusion as well as with the initial antisymmetric disturbances. Furthermore, if there is only the symmetric motion at the initial instant, namely, $\psi_S = \psi_S$, $\frac{m}{n_2} = 0$, then the antisymmetric motion and thus the asymmetric motion will form owing to the antisymmetric distributions of the surface height above sea level and of v_2 . Observations demonstrate that the symmetric component of \bar{z}_0 nearly has the same magnitude as its antisymmetric component, in particular, in the polar region or the neighboring areas of the point (90°E, 30°N) or (90°E, 30°S). $z_{0.4} \gg z_{0.5}$. Therefore, the antisymmetric motion caused by orography is of importance. However, if the effects of orography and of the horizontal diffusion are not taken into account and the motion is symmetric at the initial instant, then at any instant $t \gg t_0$, the motion would still be symmetric. This demonstrates the conclusion drawn by Liao (1980).

V. NUMERICAL EXPERIMENTS

In order to verify the previous results, three numerical experiments have been performed. In the following we shall describe them in detail.

1. Experiment 1

The Rossby-Haurwitz wave

$$\psi = -a^2 \omega \sin \varphi + a^2 K \sin \varphi \cos^m \varphi \cos m \hat{\lambda} \tag{25}$$

is taken as the initial ψ -field (Fig. 1). Then Eq. (1) in spectral form is utilized to perform time integration. Here m=4, $\omega=K=7.29\times10^{-6} {\rm s}^{-1}$, $a=6.371\times10^6 {\rm m}$. Time integration is performed up to the 58th day without considering orography, horizontal diffusion or time filtering. The results show that up to the 30th day the motion keeps the original shape well, the motion is basically symmetric and the computations obtained with the global model and those obtained with the hemispheric model are nearly the same for the same hemisphere. Fig. 2 is the result of the 30th day obtained with the global model. These not only show that the symmetry of the symmetric motion at the initial instant would be maintained in the course of time integration, but also are consistent with the previous analysis.

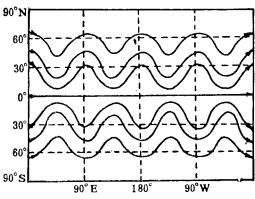


Fig. 1. The Rossby-Haurwitz wave taken as the initial ψ -field. (isolines of ψ , the same is true thereinafter).

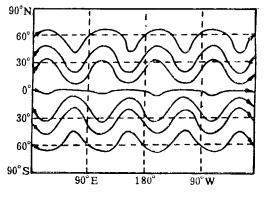


Fig. 2. The computed 30th day ψ -field in Experiment 1.

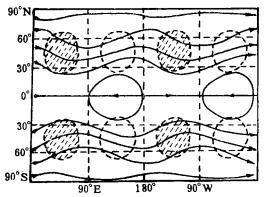


Fig. 3. The computed 72-h ψ -field in Experiment 2 (shaded areas stand for the mountains; the unshaded areas the valleys),

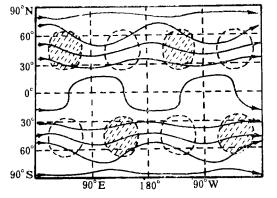


Fig. 4. The computed 72-h ψ -field in Experiment 3,

2. Experiment 2

The initial ψ -field is

$$\psi = -au_0 \left(\sin \varphi - \frac{1}{4}\sin 4\varphi\right) , \qquad (26)$$

and

$$z_0 = \hat{z}_0 \sin 2\lambda \sin^2 2\varphi \quad , \tag{27}$$

where $\hat{z}_0 = 2000$ m, $u_0 = 20$ m/s, $v_2 = 2.5 \times 10^5$ m²/s, $\hat{a} = 0.5$. It can readily be seen from Eqs. (26) and (27) that at the initial instant the motion is zonal and symmetric; the orography is symmetric too. This experiment is performed up to the 24th day. At the early days of integration the troughs appear in the leesides of the mountains and then move eastward slowly. During these days the symmetry of the motion keeps well. This demonstrates the theoretical result that the symmetric motion would maintain its symmetry even under the action of the symmetric orographical forcing; it is also consistent with the previous analysis. Here gives the computed 72-h ψ -field (Fig. 3).

3. Experiment 3

Assume the surface height above sea level to be antisymmetric, namely,

$$z_0 = \hat{z}_0 \sin 2\lambda \sin^2 2\varphi \operatorname{sgn}(\varphi) . \tag{28}$$

The initial ψ -field and the parameters involved are the same as in Experiment 2. The experiment is performed up to the 24th day. The computed 72-h ψ -field is shown in Fig.4. It can be seen that cross-equatorial flows obviously appear in the figure. Comparing the computational results by the global model with those by the hemispheric model show that their differences are quite large. This is worthy of note.

VI. DISCUSSION

In solving Eqs. (14) and (22), the cases of $\sigma_{n_1}^m = 0$ and of $\hat{\sigma}_{n_2}^m = 0$ have not been considered. In these cases it is evident that

$$\psi_{A,n_1}^{m} = i \left(\hat{a}_{n_1}^m z_S, \sum_{n_1+1}^m + \hat{b}_{n_1}^m z_S, \sum_{n_1+1}^m \right) t + \psi_{A,n_1}^m (0),$$

and

$$\psi_{S,n_2} = \{ i (\hat{a}_{n_2}^m z_{A,n_2+1} + \hat{b}_{n_2-1}^m z_{A,n_2-1}) + (\hat{c}_{n_2}^m \psi_{A,n_2+1} + \hat{d}_{n_2}^m \psi_{A,n_2-1}) \} t + \psi_{S,n_2}^m (0) .$$

Therefore, in these cases, the solutions to ψ_A and ψ_S should be rewritten as follows:

$$\psi_{A} = \sum_{m, n_{1}} \{ (\psi_{A_{i}n_{1}}^{m})_{1} + (\psi_{A_{i}n_{1}}^{m})_{2} \} e^{im\lambda} P_{n_{1}}^{m}$$
(16)

and

$$\psi_{S} = \sum_{m, n_{2}} \{ (\psi_{S}, n_{n_{2}}^{m})_{1} + (\psi_{S}, n_{n_{2}}^{m})_{2} \} e^{im\lambda} P_{n_{2}}^{m} , \qquad (24)'$$

where

$$(\psi_{A_{\ell^{n}1}}^{m})_{1} = \begin{cases} 0, & \text{when } \sigma_{n_{1}}^{m} \neq 0 \\ i(\hat{a}_{n_{1}}^{m} z_{S_{\ell^{n}1}+1} + \hat{b}_{n_{1}}^{m} z_{S_{\ell^{n}1}-1})t + \psi_{A_{\ell^{n}1}}^{m}(0), & \text{when } \sigma_{n_{1}}^{m} = 0; \end{cases}$$

$$(\psi_{A,n_{1}}^{m})_{2} = \begin{cases} \psi_{A,n_{1}}^{m}(0) e^{-i\sigma_{n_{1}}^{m}t} + \frac{1}{\sigma_{n_{1}}^{m}}(\hat{a}_{n_{1}}^{m}z_{S},_{n_{1}+1}^{m} + \hat{b}_{n_{1}}^{m}z_{S},_{n_{1}-1}^{m}) \\ & \times (1 - e^{-i\sigma_{n_{1}}^{m}t}), \text{ when } \sigma_{n_{1}}^{m} \neq 0, \\ 0, \text{ when } \hat{\sigma}_{n_{2}}^{m} \neq 0, \\ \{i(\hat{a}_{n_{2}}^{m}z_{A,n_{2}+1}^{m} + \hat{b}_{n_{2}}^{m}z_{A,n_{2}-1}^{m}) + \hat{c}_{n_{2}}^{m}\psi_{A,n_{2}+1}^{m} + \hat{d}_{n_{2}}^{m}\psi_{A,n_{2}-1}^{m}\}t \\ & + \psi_{S},_{n_{2}}^{m}(0), \text{ when } \hat{\sigma}_{n_{2}}^{m} = 0 \end{cases}$$

$$\{\psi_{S},_{n_{2}}^{m}(0) - \frac{1}{\hat{\sigma}_{n_{2}}^{m}}(\hat{a}_{n_{2}}^{m}z_{A},_{n_{2}+1}^{m} + \hat{b}_{n_{2}}^{m}z_{A},_{n_{2}-1}^{m}) \} e^{-i\hat{\sigma}_{n_{2}}^{m}t} \\ & + \frac{1}{\hat{\sigma}_{n_{2}}^{m}}(\hat{a}_{n_{2}}^{m}z_{A},_{n_{2}+1}^{m} + \hat{d}_{n_{2}}^{m}\psi_{A},_{n_{2}-1}^{m})) \} e^{-i\hat{\sigma}_{n_{2}}^{m}t} \\ & + \frac{1}{\hat{\sigma}_{n_{2}}^{m}}(\hat{a}_{n_{2}}^{m}z_{A},_{n_{2}+1}^{m} + \hat{d}_{n_{2}}^{m}\psi_{A},_{n_{2}-1}^{m})), \text{ when } \hat{\sigma}_{n_{2}}^{m} \neq 0, \\ 0, \text{ when } \hat{\sigma}_{n_{2}}^{m} = 0. \end{cases}$$

VII. THE ATMOSPHERIC INTERACTIONS BETWEEN NORTHERN AND SOUTHERN HEMI-SPHERES

As is shown by the previous analysis, if the initial motion and the distributions of the surface height above sea level and of the horizontal diffusion coefficient are all symmetric, then the motion would be symmetric forever. In this case there is no atmospheric interaction between Northern and Southern Hemispheres; furthermore, the expression

$$v=v_A=v_S=0$$
,

always holds at the equator, namely, no cross-equatorial flow occurs.

However, if there is antisymmetric orography or horizontal diffusion, then the antisymmetric motion caused by it would change the flow fields over Northern and Southern Hemispheres and their relative distribution. If the global average of u_A is greater than zero, then the zonal flow of the Northern Hemisphere is stronger than that of the Southern Hemisphere; if the global average of u_A is less than zero, then the zonal flow of the Southern Hemisphere is stronger than that of the Northern Hemisphere. And, in this case

$$v_{S} = \frac{1}{a} \sum_{m, n_{2}} \{ (\psi_{S}, m_{2}^{m})_{1} + (\psi_{S}, m_{2}^{m})_{2} \} ime^{im\lambda} P_{n_{2}}^{m}(0)$$

at the equator. Since $n_2 - m$ is an even number and $P_{n_2}^m(0) \neq 0$, in general, v_s would not vanish, thus cross-equatorial flow would occur.

From the above expression it will be seen that the magnitude and direction of the cross-equatorial flow depend upon the magnitude of α and the distributions of orography and of horizontal diffusion. In general, owing to the fact that the upslope flows in the high and middle latitudes are dominant, it seems to be able to explain the fact that the orographical effects caused by the atmospheric general circulation in those regions in the Northern or Southern Hemispheres are connected with the generation of the cross-equatorial flow, and thus with the outbreak of monsoon and so on. This also seems to be a type of teleconnection. It is understood that in the presence of the cross-equatorial flow, the exchange of momentum, energy and vorticity between Northern and the Southern Hemispheres would

happen, and the interaction between the flow fields over both hemispheres would be caused. This is worthy of further research.

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