THE INFLUENCE OF STRATIFICATION ON FRONTOGENESIS CAUSED BY GEOSTROPHIC ADJUSTMENT*

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ABSTRACT

A uniform, inviscid, incompressible fluid in a two-dimensional plane (x, z) is considered. Three principles: conservation of potential vorticity, conservation of absolute momentum, and conservation of mass are used for this study. If the initial mass field and the initial wind field do not satisfy geostrophic balance, then through geostrophic adjustment under suitable conditions, the frontogenesis will finally occur. Our work points out that the initial density distribution greatly influences the frontal features. If the stratification in cold air is the same as that in warm air, two frontogeneses will occur at top and bottom boundaries respectively. If the stratification in cold air is larger than that in warm air, the frontogenesis at the bottom boundary still exists, but the other at the top boundary disappears. This result makes us further understand the mechanism of the frontogenesis in the real atmosphere.

Key words: mesoscale weather, geostrophic adjustment, frontogenesis

I. BASIC EQUATIONS AND NUMERICAL METHODS

In the rotating earth, when the initial mass field and the initial wind field do not satisfy geostrophic balance, then through geostrophic adjustment, these two fields will finally come to geostrophic balance. Rossby (1937; 1938), Ou (1984; 1986), Van Heijst (1985), McWilliams (1988) and Glendening (1993) et al. studied the problem of geostrophic adjustment in the atmosphere and oceans, and related the adjustment with the frontogenesis. Wu and Blumen (1995), Blumen and Wu (1995) studied the frontogenesis and the adjustment problem with the zero potential vorticity flow and the uniform potential vorticity flow. They pointed out that if the initial mass field and the initial wind field do not satisfy geostrophic balance, through geostrophic adjustment, front will form, and the frontogenesis depends on the spatial scale. All above researches supposed that the stratification in cold air was the same as that in warm air. However, the temperature lapse rate γ is not a constant in the real atmosphere, and the stratification in cold air was larger

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than that in warm air. What will it be if γ is not a constant? It is a problem of practical significance and will be studied in this paper.

The basic equations are: conservation of potential vorticity, conservation of absolute momentum, and conservation of mass. Details can be found in Blumen and Wu (1995).

Introducing geostrophic momentum coordinates $X = x + \frac{v}{f}$ and $Z = z$ (Hoskins 1975), and applying the method of nondimension, these conservation equations can be rewritten as

$$q = q_0 = \left(1 - \frac{\partial v}{\partial X}\right)^{-1} \frac{\partial \rho}{\partial Z}, \tag{1}$$

$$X = X_0, \tag{2}$$

and

$$\rho(X, Z) = \rho_0(X_0, Z_0), \tag{3}$$

where q_0 is the initial potential vorticity, q the final potential vorticity, v the final velocity, ρ_0 the initial density, and ρ the final density. If the initial mass field and the initial wind field do not satisfy geostrophic balance, through geostrophic adjustment, the final state is characterized by thermal wind balance, which may be expressed as

$$\frac{\partial v}{\partial Z} = \frac{\partial \rho}{\partial X}. \tag{4}$$

Equations (1) and (4) are a closed set for variables ρ and v if q is given. Blumen and Wu (1995) studied cases with the zero potential vorticity and the uniform potential vorticity, and the present work is to study a simple case of nonuniform potential vorticity. For simplicity, q_0 is assumed to be only the function of X. Eliminating ρ or v respectively from Eqs. (1) and (4), we can get

$$\frac{\partial^2 \rho}{\partial X^2} - \frac{\partial}{\partial Z}\left(\frac{1}{q_0}\frac{\partial \rho}{\partial Z}\right) = 0, \tag{5}$$

and

$$\frac{\partial^2 v}{\partial X^2} - \frac{1}{q_0}\left(1 - \frac{\partial v}{\partial X}\right)\frac{\partial q_0}{\partial X} - \frac{1}{q_0}\frac{\partial^2 v}{\partial Z^2} = 0. \tag{6}$$

If the initial density ρ_0, the initial velocity v_0, and the initial potential voticity q_0 are given, with appropriate boundary conditions, Eqs. (5) and (6) can be solved by the method of over-relaxation-iteration. Applying the coordinate transformation, the solution in (x, z) space can be obtained finally.

The initial density distribution is given as

$$\rho_0(x_0, z_0) = -\gamma z_0 + \rho_0^*(x_0), \tag{7}$$

and

$$\rho_0^*(x_0) = -\epsilon_2 \frac{\sqrt{\pi}}{2}\mathrm{erf}(a x_0), \tag{8}$$

where $\gamma = \gamma(x_0)$ is the initial stratification parameter, ϵ_2 and a are constant parameters, and erf stands for error function. The initial velocity distribution satisfies $v_0 = 0$.

Solid boundary condition is applied at the top and the bottom, and with the conservation of ρ, then the initial density field satisfies

$$\rho(X, 0) = \rho_0(X_0, 0), \tag{9}$$

and

$$\rho(X, 1) = \rho_0(X_0, 1). \tag{10}$$

If $\gamma = 1$, this means that the stratification in cold air is the same as that in warm air. In this case, $q_0 = -1$, the potential vorticity is uniform. Under suitable initial conditions and appropriate boundary conditions, we can get the final distribution of ρ and v by solving Eqs. (5) and (6). If the initial state is not the geostrophic balance state, through geostrophic adjustment, there are two frontogeneses at top and bottom boundaries respectively in the density field, and there are two velocity jets in the wind field. Our numerical solutions are the same as the analytical solutions given by Blumen and Wu (1995). These results and figures are not shown here for simplicity.

Many observational facts show that the temperature lapse rate γ in cold air is not the same as that in warm air. It is larger in cold air than in warm air. Now we assume that the temperature lapse rate γ on the boundary surface varies continuously between cold air and warm air, i. e.

$$\gamma = -0.3 \in_2 \mathrm{erf}(\alpha x_0 + \varphi) + 1.0, \tag{11}$$

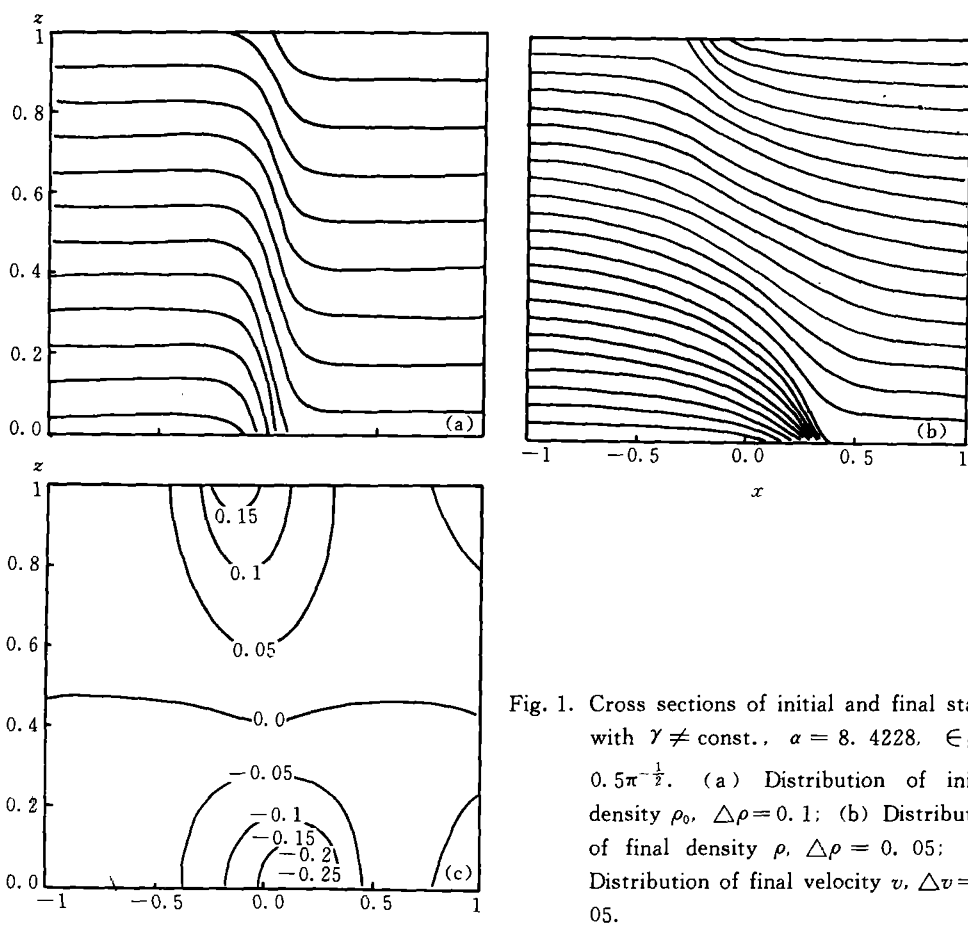

Fig. 1. Cross sections of initial and final states with $\gamma \neq$ const., $\alpha = 8.4228$, $\in_2 = 0.5\pi^{-\frac{1}{2}}$. (a) Distribution of initial density ρ_0, $\triangle\rho = 0.1$; (b) Distribution of final density ρ, $\triangle\rho = 0.05$; (c) Distribution of final velocity v, $\triangle v = 0.05$.

where φ is the phase function. In the following, φ is taken as zero, $\in_2$ and α are constants, and erf is the error function.

Similarly, under suitable initial conditions and appropriate boundary conditions, by solving Eqs. (5) and (6), we can get the final distribution of ρ and v, as in Fig. 1.

Figure 1a is the distribution of the initial density ρ_0. It shows that the stratification in cold air is larger than that in warm air. Figure 1b is the distribution of the final density ρ. It shows that the frontogenesis at the bottom boundary still exists, but the frontogenesis at the top boundary disappears. Figure 1c is the distribution of the final velocity v. Contrasted with the case $\gamma = 1$, it shows that the velocity jets still appear, and the intensity of the velocity jet in the low air remains unchanged while the intensity of the velocity jet in the upper air is obviously getting weak. Either Hoskins (1975), or Blumen and Wu (1995) pointed out that the critical condition for frontogenesis is $\partial v/\partial X = 1$ according to the relationship of coordinate transformation. Here we give out two curves of $\partial v/\partial X$ varying with x at the top boundary and the bottom boundary respectively in Fig. 2.

From Fig. 2, we can find that $\partial v/\partial X$ equals 1.0 at a special point at the bottom boundary, where a frontogenesis occurs. The position of the front here is the same as the position in Fig. 1b. At the top boundary, because $\partial v/\partial X$ is always less than 1.0, therefore, there is no any frontogenesis. This result can be explained from physical concepts as follows.

In the lower atmosphere, the horizontal density gradient is large. When air moves from the region where the density is large to the region where the density is small, it is

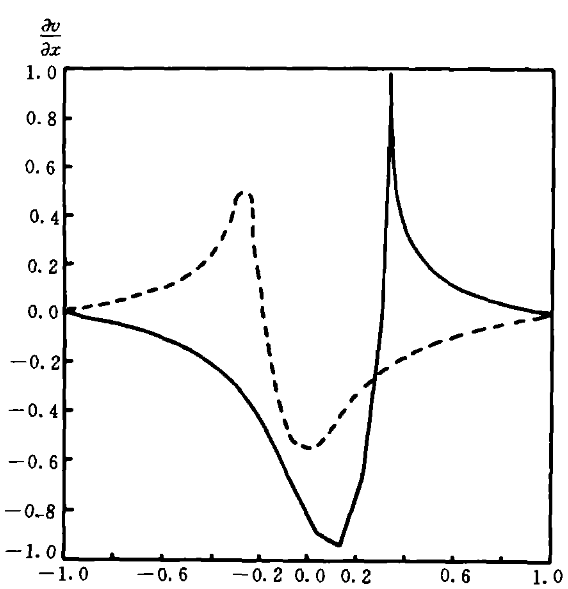

Fig. 2. Curves of $\partial v/\partial X = 1$ varying with x when $\gamma \neq$ const. Solid line denotes the curve at bottom boundary, dotted line denotes the curve at the top boundary. Frontogenesis occurs at $\partial v/\partial X = 1$.

easy to form the accumulation of mass. So it is easy to form a front at the bottom boundary. In the upper air, due to the effect of compensated flow, air moves from the region where the density is small to the region where the density is large. Because the horizontal density gradient in the upper air is smaller than that in the lower part of air, it is not as easy as that in the lower atmosphere to form the accumulation of mass. This gives the reason why the collapse of isopycnals in the lower boundary appears but no collapse in the upper boundary.

II. CONCLUSION

If the initial mass field and the initial wind field do not satisfy geostrophic balance, through geostrophic adjustment, these two fields will finally come to geostrophic balance. And the frontogenesis will occur. The distribution of the initial density field influences the frontal features greatly. If the initial stratification in cold air is the same as that in warm air, two frontogeneses will occur simultaneously at top and bottom boundaries respectively, and two velocity jets will appear in the wind field. The frontogenesis at the bottom boundary is due to the local accumulation of mass. The one at the top boundary, which is seldom seen in the real atmosphere, is due to the compensative effect of mass flow. Many observational facts show that the temperature lapse rate γ in cold air is not the same as that in warm air. It is larger in cold air than in warm air. Considering the difference of these stratifications, the frontogenesis at the bottom boundary still exists, while the other at the top boundary disappears. The nonuniform of stratification in essence is the nonuniform of potential vorticity.

The above example is only a simple case with the nonuniform potential vorticity. The result will make us further understand the mechanism of frontogenesis in the real atmosphere. It is more complicated for other cases with the nonuniform potential vorticity $q=q(x,z)$.

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