THE FOUNDATION AND MOVEMENT OF TROPICAL SEMI-GEOSTROPHIC ADAPTATION

Chao Jiping (巢纪平)

National Research Center for Marine Environment Forecasts, Beijing 100081

and *Lin Yonghui* (林永辉)

Institute of Atmospheric Physics, Chinese Academy of Sciences, Beijing 100029

Received July 18, 1995; revised October 24, 1995

ABSTRACT

The breakdown and foundation of geostrophic balance is one of the important movements in the mid- and high-latitude atmosphere and oceans. In the tropical area, the value of Coriolis parameter is so small that it is difficult to satisfy the bi-geostrophic equilibrium between the pressure and velocity fields. However, in the tropical area, the zonal velocity of some motions in the atmosphere and oceans is large, so the Coriolis force is not small, geostrophic balance can exist in zonal direction, i. e. semi-geostrophic balance. Furthermore, in the dominant area of Hadley circulation in the atmosphere or the area near the ocean meridional boundary, the meridional velocity is large, so geostrophic balance can also exist in meridional direction. In this paper, the process of the dispersion of inertial gravity wave and the foundation of semi-geostrophic balance are first discussed. Second, the adjustment process between the velocity and pressure fields after adaptation is also viewed, and the scale criterion of the semi-geostrophic adaptation is discussed, i. e. for the motion with meridional scale greater than the equatorial Rossby radius of deformation, the velocity and pressure fields after adaptation change to fit the initial pressure field; on the contrary, the fields change to fit the initial zonal velocity field, and the strength of the fields after adaptation depends on the zonal scale.

Key words: dispersion of inertial gravity wave, semi-geostrophic adaptation, scale criterion

I. INTRODUCTION

For the motions of atmosphere and ocean of a rotating earth, if the change of Coriolis parameter f ($=2\Omega\sin\varphi$, Ω is the angular velocity of the rotating earth, φ is the latitude) with the latitude (i. e. β-effect) is ignored, and the initial equilibrium is disturbed, then inertial gravity wave will be stimulated, with the dispersion of the wave, a new geostrophic balance will be founded through the mutual adjustment and adaptation between the pressure and velocity fields. The process of the dispersion of inertial gravity wave and the foundation of geostrophic balance is known as the geostrophic adaptation (the field of adaptation follows an invariant of potential vorticity). The movement under the geostrophic balance is called evolution movement. Since Rossby (1937; 1938) first put forward the concept, there have been many advances, including the contributions of Yeh (1957) and Zeng (1963). So to speak, the geostrophic adaptation of mid- and high-latitudes is basically clear (Yeh and Li 1965).

In the tropical area, the Coriolis parameter is so small that it is very difficult to satisfy the bi-geostrophic balance in both zonal and meridional directions, simultaneously. The geostrophic balance often exists in one direction, for example, the changes of physical field of atmosphere and ocean mainly emerge in a narrow equatorial area whose width is about one Rossby radius of deformation, and the movements along the zonal direction are uniform and the strength is strong, so it is possible for the geostrophic balance to exist in zonal direction, and generally we name it the long-wave approximation or low-frequency approximation. On the other hand, in the area near ocean meridional boundary, the meridional velocity along the boundary and the pressure gradient vertical to the boundary are large, they approximately satisfy geostrophic balance. Similarly, in the dominant area of Hadley circulation in the atmosphere, the movement also satisfies geostrophic balance in meridional direction, the change of physical field of this movement along the zonal direction is large or the wave numbers are big, so it is called short wave approximation. Since the geostrophic balance generally exists in one direction in the tropical area, the balance is named semi-geostrophic balance, the founding process of semi-geostrophic balance is known as semi-geostrophic adaptation, the movement after semi-geostrophic adaptation is named semi-geostrophic evolution movement.

In this paper, the process of the dispersion of inertial gravity wave and the foundation of semi-geostrophic balance in the tropical area is viewed, and at the same time, the mutual adjustment of the fields under the semi-geostrophic adaptation is also discussed.

II. THE EQUATIONS OF MOTION

The linearized shallow-water equations in the equatorial β-plane are

$$\frac{\partial u}{\partial t} - \beta y v + \frac{\partial \varphi}{\partial x} = 0, \tag{1}$$

$$\frac{\partial v}{\partial t} + \beta y u + \frac{\partial \varphi}{\partial y} = 0, \tag{2}$$

$$\frac{\partial \varphi}{\partial t} + C^2\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0, \tag{3}$$

where u and v represent the eastward (x) and northward (y) components of velocity, respectively. φ is the geopotential height of the atmosphere, for the ocean $\varphi = g'\eta$, η is the disturbed depth of the thermocline, and

$$g' = \frac{\rho_2 - \rho_1}{\rho_2} g, \tag{4}$$

where g is the acceleration of gravity, g' is the reduced gravity. The ocean is separated into two layers with density ρ_1 and ρ_2, respectively; the lower-layer with density ρ_2 is motionless. The corresponding speed of gravity wave is

$$C = (g'H)^{1/2}, \tag{5}$$

where H is the depth of the upper-layer. Generally, for the baroclinic fluid, the motion may be expanded in terms of eigenmodes in vertical direction. For any mode, we have

$$C = (gh)^{1/2}, \tag{6}$$

where h is the equivalent depth of the eigenmode, $\varphi = gh$. In this case, Eqs. (1) — (3) are the horizontal structure equations of the baroclinic fluid corresponding to certain vertical eigenmode.

Taking the time scale $(2\beta C)^{-1/2}$, the horizontal scale $(C/2\beta)^{1/2}$, the velocity scale C, and the geopotential height scale C^2, the forms of non-dimensional equations can be written as

$$\in_1 \frac{\partial u}{\partial t} - \frac{1}{2} yv + \frac{\partial \varphi}{\partial x} = 0, \tag{7}$$

$$\in_2 \frac{\partial v}{\partial t} + \frac{1}{2} yu + \frac{\partial \varphi}{\partial y} = 0, \tag{8}$$

$$\in_3 \frac{\partial \varphi}{\partial t} + \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{9}$$

where $\in_i (i=1, 2, 3)$ is the sign symbol, it takes 1 or 0. When taking 0, Eqs. (7) — (9) represent the meridional geostrophic balance, zonal geostrophic balance and non-divergent motion, respectively.

Eqs. (7) — (9) give

$$\in_1 \in_3 \frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = \in_3 \frac{1}{2} y \frac{\partial v}{\partial t} + \frac{\partial^2 v}{\partial x \partial y}, \tag{10}$$

$$\in_2 \in_3 \frac{\partial^2 v}{\partial t^2} - \frac{\partial^2 v}{\partial y^2} = - \in_3 \frac{1}{2} y \frac{\partial u}{\partial t} + \frac{\partial^2 u}{\partial x \partial y}. \tag{11}$$

Eliminating u from Eqs. (10) and (11) gives

$$\in_1 \in_2 \in_3 \frac{\partial^3 v}{\partial t^3} - (\in_2 \frac{\partial^2}{\partial x^2} + \in_1 \frac{\partial^2}{\partial y^2} - \in_3 \frac{1}{4} y^2) \frac{\partial v}{\partial t} - \frac{1}{2} \frac{\partial v}{\partial x} = 0. \tag{12}$$

Eq. (12) is Matsuno (1966) equation describing the tropical trapped waves, including high-frequency inertial gravity wave, low-frequency Rossby wave, inertial gravity-Rossby mixed wave and non-dispersion Kelvin wave. Figure 1 gives the dispersion curves of various waves, including the westward propagating boundary-wave with damping amplitude.

III. THE DISPERSION OF INERTIAL GRAVITY WAVE

Ignoring β-effect, Eq. (12) becomes

$$\in_1 \in_2 \in_3 \frac{\partial^3 v}{\partial t^3} - (\in_2 \frac{\partial^2}{\partial x^2} + \in_1 \frac{\partial^2}{\partial y^2} - \in_3 \frac{1}{4} y^2) \frac{\partial v}{\partial t} = 0. \tag{13}$$

Integrating (13) over the time gives

$$\in_1 \in_2 \in_3 \frac{\partial^2 v}{\partial t^2} - (\in_2 \frac{\partial^2}{\partial x^2} + \in_1 \frac{\partial^2}{\partial y^2} - \in_3 \frac{1}{4} y^2) v = [\in_1 \in_2 \in_3 \frac{\partial^2 v}{\partial t^2} - (\in_2 \frac{\partial^2}{\partial x^2} + \in_1 \frac{\partial^2}{\partial y^2} - \in_3 \frac{1}{4} y^2) v]_{t=0}. \tag{14}$$

Since the initial motion may be non-geostrophic and divergent, the motion is still controlled by Eqs. (7), (8), (10) and (11). Considering Eqs. (11) and (7), (14) becomes

$$\in_1 \in_2 \in_3 \frac{\partial^2 v}{\partial t^2} - (\in_2 \frac{\partial^2}{\partial x^2} + \in_1 \frac{\partial^2}{\partial y^2} - \in_3 \frac{1}{4} y^2) v = \frac{\partial}{\partial x} [\in_3 \frac{1}{2} y \varphi - (\in_2 \frac{\partial v}{\partial x} - \in_1 \frac{\partial u}{\partial y})]_{t=0}. \tag{15}$$

Ignoring the sign symbol, Eq. (15) becomes

$$\frac{\partial^2 v}{\partial t^2} - (\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} - \frac{1}{4} y^2) v = F(x, y, 0), \tag{16}$$

where

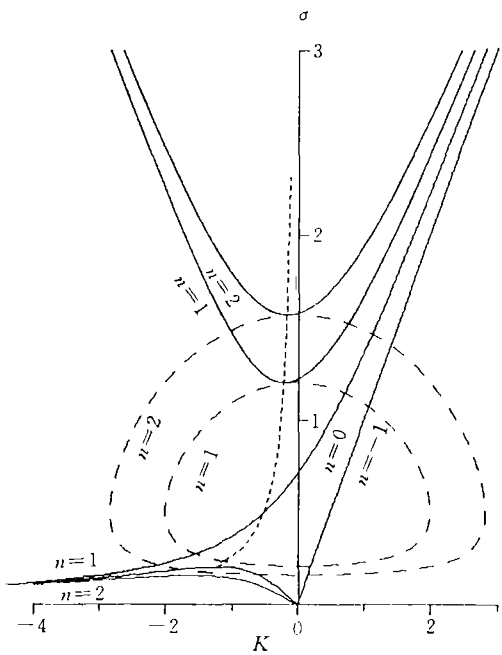


Fig. 1. Dispersion curves for equatorial waves. The vertical axis is the frequency and the horizontal axis is the east-west wavenumber. The curve labeled 0 corresponds to the mixed Rossby-gravity wave. The upper curves labeled 1 and 2 are the first two gravity wave modes and the corresponding lower curves are the first two Rossby wave modes. The straight line labeled −1 corresponds to the Kelvin wave. The dashed curves are the boundary waves.

$$F(x,y,0) = \frac{\partial}{\partial x}\left[\frac{1}{2}y\varphi - \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\right]_{t=0}. \tag{17}$$

Eq. (16) describing the dispersion of inertial gravity wave is resolved under the following conditions

$$t = 0, \qquad v = \Phi_1(x,y), \qquad \frac{\partial v}{\partial t} = \Phi_2(x,y), \tag{18}$$

$$|x| \to \infty, \qquad v \to 0. \tag{19}$$

Variables v, F, Φ_1, Φ_2 are expanded in terms of the parabolic cylinder functions (i. e. Weber functions):

$$v = \sum_n v_n(x,t)D_n(y), \tag{20a}$$

$$F = \sum_n F_n(x)D_n(y), \tag{20b}$$

$$\Phi_1 = \sum_n \Phi_{1n}(x)D_n(y), \tag{20c}$$

$$\Phi_2 = \sum_n \Phi_{2n}(x)D_n(y). \tag{20d}$$

Therefore the coefficients of the Weber functions satisfy

$$\frac{\partial^2 v_n}{\partial t^2} - \frac{\partial^2 v_n}{\partial x^2} + (n + \frac{1}{2})v_n = F_n \tag{21}$$

and

$$t = 0, \quad v_n = \Phi_{1n}, \quad \frac{\partial v_n}{\partial t} = \Phi_{2n}, \tag{22}$$

$$|x| \to \infty, \quad v_n \to 0. \tag{23}$$

Using Fourier method or Laplance transformation, the solution of Eq. (21) satisfying conditions (22) and (23) is

$$\begin{aligned} v_n = & \frac{1}{2}\int_{-\infty}^{\infty} \Phi_{1n}(x') \frac{J_1\left[\sqrt{n+\frac{1}{2}}\ \sqrt{t^2-(x-x')^2}\right]}{\sqrt{t^2-(x-x')^2}} \mathrm{d}x' \\ & + \frac{1}{2}\int_{-\infty}^{\infty} \Phi_{2n}(x') J_0\left[\sqrt{n+\frac{1}{2}}\ \sqrt{t^2-(x-x')^2}\right] \mathrm{d}x' \\ & + \frac{1}{2}\int_0^t\int_{-\infty}^{\infty} F_n(x') J_0\left[\sqrt{n+\frac{1}{2}}\ \sqrt{t'^2-(x-x')^2}\right] \mathrm{d}x'\mathrm{d}t', \end{aligned} \tag{24}$$

where J_ν is Bessel function of order ν. Using the properties of Bessel function, i. e., when time is longer, the influence due to the initial conditions disappears quickly, then the main contribution of (24) is

$$v_n = \frac{1}{2}\int_0^t\int_{-\infty}^{\infty} F_n(x') J_0\left[\sqrt{n+\frac{1}{2}}\ \sqrt{t'^2-(x-x')^2}\right] \mathrm{d}x'\mathrm{d}t'. \tag{25}$$

Derivation of (25) with respect to the time is

$$\frac{\partial v_n}{\partial t} = \frac{1}{2}\int_{-\infty}^{\infty} F_n(x') J_0\left[\sqrt{n+\frac{1}{2}}\ \sqrt{t^2-(x-x')^2}\right] \mathrm{d}x'. \tag{26}$$

Now, continuously evaluate the solution as the time increases greatly. If the value of F_n focuses on the original point, then assume

$$F_n(x) = A_n\delta(x), \tag{27}$$

where $\delta(x)$ is Delta function. Considering (27), Eqs. (25) and (26) respectively become

$$v_n = \frac{A_n}{2}\int_0^t J_0\left[\sqrt{n+\frac{1}{2}}\ \sqrt{t'^2-x^2}\right] \mathrm{d}t', \tag{28}$$

$$\frac{\partial v_n}{\partial t} = \frac{A_n}{2} J_0\left[\sqrt{n+\frac{1}{2}}\ \sqrt{t^2-x^2}\right]. \tag{29}$$

Note that in the area of $t > x$, when $t \to \infty$, Eq. (29) gives

$$\frac{\partial v_n}{\partial t} \sim \mathrm{O}(t^{-\frac{1}{2}}), \tag{30}$$

i. e. the time derivative of meridional velocity decays according to $t^{-1/2}$, but the meridional velocity does not equal zero. In fact, near the original point ($x=0$), Eq. (28) gives

$$v_n(0, t \to \infty) = \frac{A_n}{2}\int_0^{\infty} J_0\left[\sqrt{n+\frac{1}{2}}t'\right] \mathrm{d}t' = \frac{A_n}{2\sqrt{n+\frac{1}{2}}}, \tag{31}$$

which denotes that the meridional velocity is definite, and the value is small with higher n, i. e. the lower-order mode plays an important role. When $n=1$, the change of v_1/A_1 with the time is given in Fig. 2, showing that the value is almost the same as the theoretical value 0. 408 of Eq. (31) as the time equals 20.

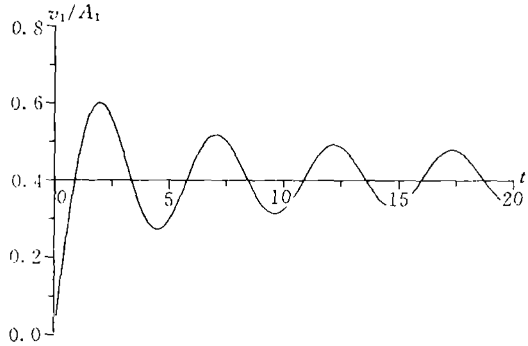


Fig. 2. Solution v_1 of Eq. (28) at $n=1$. The vertical axis is the meridional velocity in units of A_1. The horizontal axis is the time.

IV. THE FOUNDATION OF SEMI-GEOSTROPHIC BALANCE

As the time is longer, after the dispersion of inertial gravity wave, since the time derivative of meridional velocity almost equals zero, then Eq. (8) becomes

$$\frac{1}{2}yu = -\frac{\partial \varphi}{\partial y}, \tag{32}$$

i. e. the motion satisfies the zonal geostrophic balance. In fact, the foundation of zonal geostrophic balance is equavelent to $\in_2=0$. Ignoring the sign symbol, the meridional velocity after adaptation satisfies

$$-\left(\frac{\partial^2}{\partial y^2} - \frac{1}{4}y^2\right)v = \frac{\partial}{\partial x}\left[\frac{1}{2}y\varphi - \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\right]_{t=0}. \tag{33}$$

Noticeably, the initial motion may be non-geostrophic and divergent, so the sign symbol of the right-hand side of (15) should be taken as l.

Eq. (33) shows that the meridional velocity is directly given by (33), when the geostrophic balance is founded and the inertial gravity wave completely dispersed. Eq. (33) also denotes that the meridional velocity after adaptation is determined by the gradient of initial potential vorticity along the zonal direction.

Note that the zonal geostrophic balance can remove the inertial gravity wave, and make the fluid fields mutual adaptation. The zonal geostrophic balance is not a unique method to exclude the inertial gravity wave. In fact, from Eq. (15), when meridional geostrophic balance exists between the velocity and pressure fields, i. e.

$$\frac{1}{2}yv = \frac{\partial \varphi}{\partial x}, \tag{34}$$

the inertial gravity wave is also filtered, and the fields are in mutul adaptation. Similar to the previous deduction, the zonal velocity after adaptation is given by

$$-\left(\frac{\partial^2}{\partial x^2} - \frac{1}{4}y^2\right)u = \frac{\partial}{\partial y}\left[\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) - \frac{1}{2}y\varphi\right]_{t=0}, \tag{35}$$

showing that the zonal velocity after adaptation is determined by the gradient of initial potential vorticity along meridional direction.

V. THE INVARIANT OF SEMI-POTENTIAL VORTICITY

In the motion of geostrophic adaptation of the mid- and high-latitudes, the physical field after adaptation follows an invariant of potential vorticity (Obukhov 1949) governing the motion after adaptation. Now, the corresponding laws in tropical semi-geostrophic adaptation are discussed.

For the zonal semi-geostrophic motion, based on Eqs. (7) and (9), and considering Eq. (32), we obtain

$$\frac{\partial^2 u}{\partial x \partial y} + \frac{1}{2} y \frac{\partial \varphi}{\partial x} = -\left(\frac{\partial^2 v}{\partial y^2} - \frac{1}{4} y^2 v\right). \tag{36}$$

Substituting Eqs. (32) and (33) into Eq. (36), and ignoring the term associated with the change of y in order to exclude β-effect, we yield

$$\frac{\partial}{\partial x}\left[\frac{\partial^2 \varphi}{\partial y^2} - \frac{1}{4} y^2 \varphi\right] = \frac{\partial}{\partial x}\left[\frac{1}{2} y\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) - \frac{1}{4} y^2 \varphi\right]_{t=0}. \tag{37}$$

Integrating (37) with respect to x becomes

$$\frac{\partial^2 \varphi}{\partial y^2} - \frac{1}{4} y^2 \varphi = \frac{1}{2} y\left[\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) - \frac{1}{2} y \varphi\right]_{t=0}, \tag{38}$$

denoting that the inside bracket of the right-hand side is initial potential vorticity, so the adaptation field of the left-hand side is potential vorticity under the long-wave approximation. Due to the lack of x-derivatives of order two in the vorticity, Eq. (38) is named the invariant of semi-potential vorticity.

Similarly, for the meridional semi-geostrophic motion, from Eqs. (8), (9) and (34), we obtain

$$\frac{\partial^2 u}{\partial x^2} - \frac{1}{4} y^2 u = -\frac{\partial^2 v}{\partial x \partial y} + \frac{1}{2} y \frac{\partial \varphi}{\partial y}. \tag{39}$$

Substituting Eq. (35) into Eq. (39) becomes

$$\frac{\partial}{\partial y}\left[\frac{\partial v}{\partial x} - \frac{1}{2} y \varphi\right] = \frac{\partial}{\partial y}\left[\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) - \frac{1}{2} y \varphi\right]_{t=0}. \tag{40}$$

Integrating (40) with respect to y yields

$$\frac{\partial v}{\partial x} - \frac{1}{2} y \varphi = \left[\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) - \frac{1}{2} y \varphi\right]_{t=0}. \tag{41}$$

Substituting Eq. (34) into Eq. (41) gives

$$\frac{\partial^2 \varphi}{\partial x^2} - \frac{1}{4} y^2 \varphi = \frac{1}{2} y\left[\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) - \frac{1}{2} y \varphi\right]_{t=0}. \tag{42}$$

VI. THE SCALE CRITERION OF ADAPTATION FIELD

The study of geostrophic adaptation of mid- and high-latitudes shows that the process of mutual adaptation between the fields depends on the scale of motion, for the large-scale initial disturbance, the velocity field changes to fit the pressure field; for the small-scale initial disturbance, the pressure field changes to fit the velocity field. Now, the scale-criterion in tropical semi-geostrophic adaptation is viewed.

For the zonal semi-geostrophic motion, if knowing the initial potential vorticity, then the pressure field is given by (38), the zonal velocity is given by (32), and the meridional velocity is given by (33).

Let

$$t=0,\quad \begin{cases} v=0, \\ u=u^0 e^{-\alpha_1 x^2} e^{-\alpha_2 y^2}, \\ \varphi=\varphi^0 e^{-\alpha_1 x^2} e^{-\alpha_2 y^2}, \end{cases} \tag{43}$$

where

$$\alpha_1=(L_0/L_1)^2,\quad \alpha_2=(L_0/L_2)^2, \tag{44}$$

$L_0=(C/2\beta)^{1/2}$ is the equatorial Rossby radius of deformation. L_1 and L_2 are zonal and meridional characteristic scale of initial perturbation, respectively.

Substituting (43) into (33) and (38) gives

$$\left(\frac{\partial^2}{\partial y^2}-\frac{1}{4}y^2\right)v=\alpha_1(\varphi^0-4\alpha_2 u^0)xe^{-\alpha_1 x^2}ye^{-\alpha_2 y^2}, \tag{45}$$

$$\left(\frac{\partial^2}{\partial y^2}-\frac{1}{4}y^2\right)\varphi=\frac{1}{4}(4\alpha_2 u^0-\varphi^0)e^{-\alpha_1 x^2}y^2 e^{-\alpha_2 y^2}. \tag{46}$$

Variables u, v, φ are expanded in terms of the parabolic cylinder functions, i. e. Weber functions:

$$(u,v,\varphi)=\sum_{n=0}^{\infty}(u_n,xv_n,\varphi_n)e^{-\alpha_1 x^2}D_n(y). \tag{47}$$

Substituting (47) into (45), (46), (32), using the properties of Weber functions, and taking the lowest order, we obtain

$$\begin{cases} \varphi_0=\dfrac{\sqrt{2}}{8}(\varphi^0-4\alpha_2 u^0)\left(\alpha_2+\dfrac{1}{4}\right)^{-3/2}, \\ v_1=\dfrac{-\sqrt{2}}{6}(\varphi^0-4\alpha_2 u^0)\alpha_1\left(\alpha_2+\dfrac{1}{4}\right)^{-3/2}, \\ u_0=\varphi_0. \end{cases} \tag{48}$$

From (48), as $\alpha_2=(L_0/L_2)^2\ll 1$, the pressure field is more important than the zonal velocity field for the initial disturbance, i. e. the pressure, zonal and meridional velocity fields after adaptation change to fit the initial pressure field. As $\alpha_2=(L_0/L_2)^2\gg 1$, the zonal velocity field is more important than the pressure field for the initial disturbance, and the fields after adaptation change to fit the initial zonal velocity field. Furthermore, the strength of the meridional velocity after adaptation depends on α_1. As α_1 is small (i. e. the initial zonal scale is large), the strength of the meridional velocity is small.

In computation, let the non-dimensional variable $\varphi^0=u^0=1$. Substituting (48) into (47), we obtain the structure fields after zonal semi-geostrophic adaptation.

From Fig. 3, for the large scale initial disturbance (i. e. α_2 is small), the value of the pressure after zonal semi-geostrophic adaptation changes a little, but the velocity changes greatly, i. e. the velocity field changes to fit the initial pressure field. On the other hand, for the small scale initial disturbance (i. e. α_2 is large), with the increase of the value of α_2, the pressure and velocity fields after adaptation are close to zero, the value of the pressure changes a lot, and the velocity changes a little, i. e. the pressure field changes to fit the initial velocity field. When the scale of the initial disturbance equals the Rossby radius of deformation (i. e. α_2 equals 1), the values of the pressure and velocity after adaptation both change greatly. The conclusions agree with the scale-criterion of the geostrophic adaptation in mid- and high-latitude area.

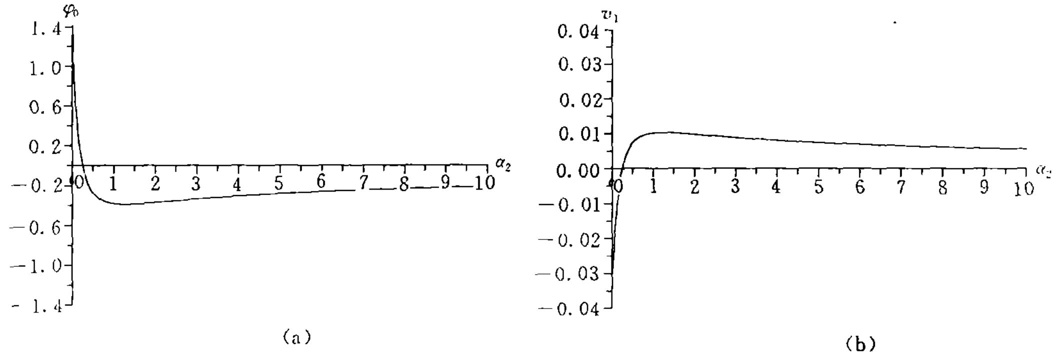


Fig. 3. The relation of the variations of the fields after zonal semi-geostrophic adaptation with the scale α_2 of initial disturbance. (a) The variations of the pressure. (b) The variations of the meridional velocity. In the computation, $\alpha_1=0.02$.

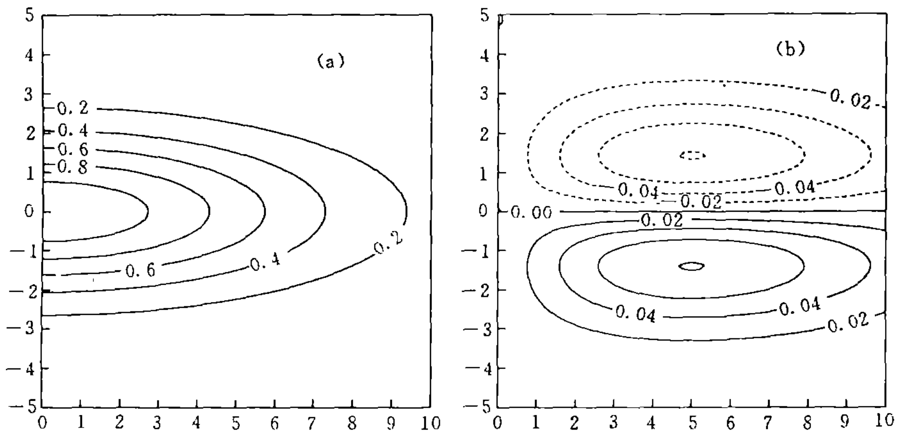


Fig. 4. The structure of the physical fields after zonal semi-geostrophic adaptation. The vertical axis is meridional length, the horizontal axis is zonal length. (a) The pressure field. (b) The meridional velocity field. In the computation, $\alpha_1=\alpha_2=0.02$.

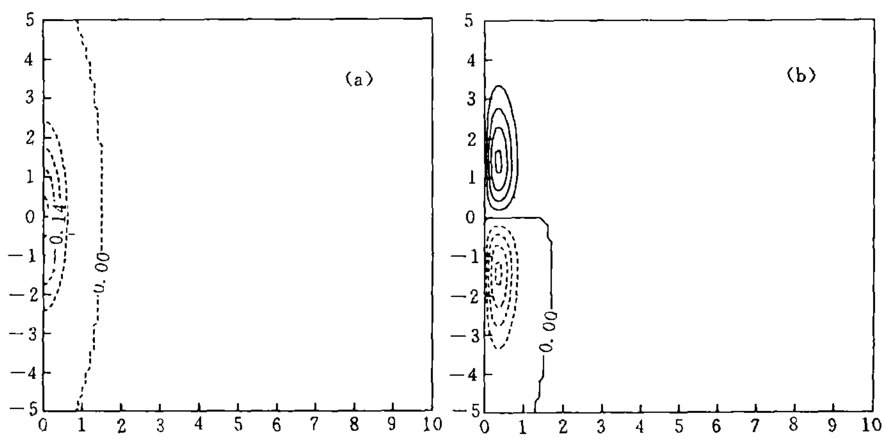


Fig. 5. As in Fig. 4 but $\alpha_1=\alpha_2=4.0$. The interval is 0.07 in diagram b.

Figures 4 and 5 give the structure fields of the velocity and pressure fields after zonal semi-geostrophic adaptation. For the large-scale initial disturbance, the velocity field and the pressure field exist in large area, and the pressure field changes a little, i.e. the velocity field changes to fit the pressure field (Fig. 4). For the small-scale initial disturbance, the velocity field and the pressure field exist in the local area, the pressure field changes greatly, i.e. the pressure field changes to fit the velocity field (Fig. 5).

If the motion is meridional geostrophic balance, then after adaptation, the pressure field is given by (42), the meridional velocity is given by (34) and the zonal velocity is given by (35).

Substituting (43) into (42) and (35) gives

$$\frac{\partial^2 \varphi}{\partial x^2} - \frac{1}{4}y^2\varphi = (\alpha_2 u^0 - \frac{1}{4}\varphi^0)y^2 e^{-\alpha_1 x^2} e^{-\alpha_2 y^2}, \tag{49}$$

$$\frac{\partial^2 u}{\partial x^2} - \frac{1}{4}y^2 u = (\frac{1}{2}\varphi^0 - 2\alpha_2 u^0)(1 - 2\alpha_2 y^2)e^{-\alpha_1 x^2} e^{-\alpha_2 y^2}. \tag{50}$$

Variables u, v, φ are expanded in terms of Weber functions, i.e.

$$(u, v, \varphi) = \sum_{n=0}^{\infty}(u_n, v_n, \varphi_n)D_n(y). \tag{51}$$

Substituting (51) into (49), (50), (34), considering the properties of Weber functions, taking the lowest order and using Green functions, the solutions of Eqs. (49), (50) and (34) become

$$\begin{cases} \varphi_0(x) = -\beta_1[(1 + \mathrm{erf}(x_1))e^{-\frac{1}{2}x} + (1 - \mathrm{erf}(x_2))e^{\frac{1}{2}x}], \\ u_0(x) = -\beta_2[(1 + \mathrm{erf}(x_1))e^{-\frac{1}{2}x} + (1 - \mathrm{erf}(x_2))e^{\frac{1}{2}x}], \\ v_1(x) = \beta_1[(1 + \mathrm{erf}(x_1))e^{-\frac{1}{2}x} - (1 - \mathrm{erf}(x_2))e^{\frac{1}{2}x}], \end{cases} \tag{52}$$

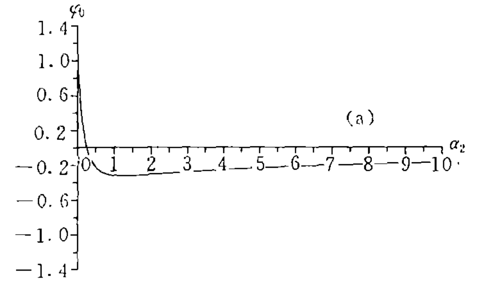
where

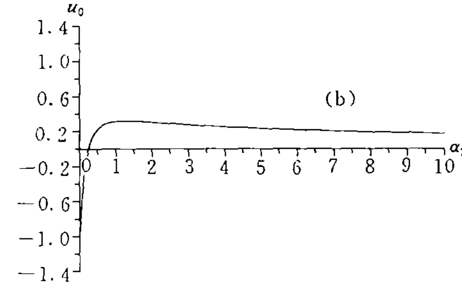
$$\begin{cases} \beta_1 = \dfrac{\sqrt{2\pi}}{16}(\alpha_2 u^0 - \dfrac{1}{4}\varphi^0)e^{\frac{1}{16\alpha_1}} \cdot \alpha_1^{-1/2} \cdot (\alpha_2 + \dfrac{1}{4})^{-3/2}, \\ \beta_2 = \dfrac{\sqrt{2\pi}}{16}(\varphi^0 - 4\alpha_2 u^0)e^{\frac{1}{16\alpha_1}} \cdot \alpha_1^{-1/2}[(\alpha_2 + \dfrac{1}{4})^{-1/2} - \alpha_2(\alpha_2 + \dfrac{1}{4})^{-3/2}], \\ x_1 = \sqrt{\alpha_1}\,(x - \dfrac{1}{4\alpha_1}), \\ x_2 = \sqrt{\alpha_1}\,(x + \dfrac{1}{4\alpha_1}), \\ \mathrm{erf}(\xi) = \dfrac{2}{\sqrt{\pi}}\displaystyle\int_0^{\xi} e^{-\tau^2}d\tau \quad \text{is error function.} \end{cases} \tag{53}$$

From Eqs. (52) and (53), as $\alpha_2 = (L_0/L_2)^2 \ll 1$, the pressure field is more important than the zonal velocity field for the initial disturbance, i.e. the pressure, zonal and meridional velocity fields after adaptation change to fit the initial pressure field. As $\alpha_2 = (L_0/L_2)^2 \gg 1$, the zonal velocity field is more important than the pressure field for the initial disturbance, and the fields after adaptation change to fit the initial zonal velocity field. Furthermore, the strength of the fields after adaptation depends on α_1, as α_1 is small (i.e. the initial zonal scale is large), and the strength of the fields is large.

In the computation, let $\varphi^0 = u^0 = 1$. Substituting (52) into (51) obtains the structure fields after meridional semi-geostrophic adaptation.

From Fig. 6, for the large scale initial disturbance (i.e. α_2 is small), the value of the





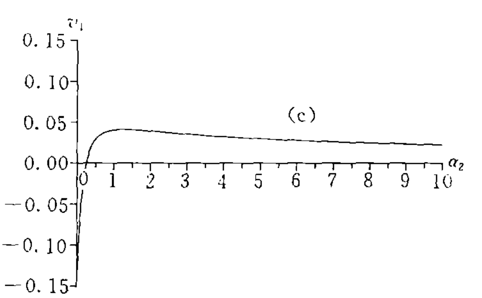


Fig. 6. The relation of the variations of the fields after meridional semi-geostrophic adaptation with the scale α_2 of initial disturbance. (a) The variations of the pressure. (b) The variations of the zonal velocity. (c) The variations of the meridional velocity. In the computation, $x=2.0$, $\alpha_1=0.02$.

pressure after meridional semi-geostrophic adaptation changes a little, but the velocity changes greatly, i. e. the velocity field changes to fit the pressure field. On the contrary, for the small scale initial disturbance (i. e. α_2 is large), with the increase of α_2, the pressure and velocity fields after adaptation are close to zero, the value of the pressure changes greatly, and the velocity changes a little, i. e. the pressure field changes to fit the initial velocity field. When the scale of the initial disturbance equals the Rossby radius of deformation (i. e. α_2 equals 1), the values of the pressure and velocity after adaptation both change greatly. The conclusions agree with the scale-criterion of the geostrophic adaptation in mid- and high-latitude area.

Figures 7 and 8 give the pressure field and meridional velocity field after meridional semi-geostrophic adaptation. For the large-scale initial disturbance, the velocity field and the pressure field also exist in large area, and the velocity field changes a lot, i. e. the velocity field changes to fit the pressure field (Fig. 7). For the small-scale initial disturbance, the velocity and pressure fields both exist in the local area, and the pressure field changes greatly, i. e. the pressure field changes to fit the velocity field (Fig. 8).

VII. CONCLUSIONS

In the tropical area, after the dispersion of an inertial gravity wave, the zonal or meridional semi-geostrophic balance can be founded, it follows an invariant of semi-potential vorticity. Based on the invariant, the author points out that the velocity and pressure fields after adaptation change to fit the initial pressure field for the large meridional-scale initial disturbance, and the fields change to fit the initial zonal velocity field for the small meridional-scale initial disturbance, and the strength of the fields after adaptation depends on the zonal scale.

In this article, taking zero as the value of the initial meridional velocity field; for the case that meridional velocity is not zero, another paper will discuss it.

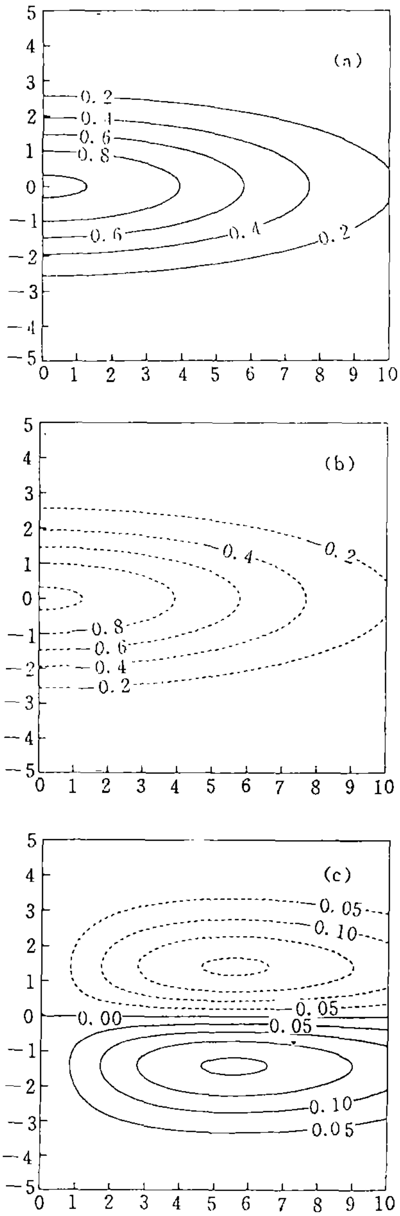


Fig. 7. The structure of the physical fields after meridional semi-geostrophic adaptation. The vertical axis is meridional length, the horizontal axis is zonal length. (a) The pressure field. (b) The zonal velocity field. (c) The meridional velocity field. In the computation, $\alpha_1 = \alpha_2 = 0.02$.

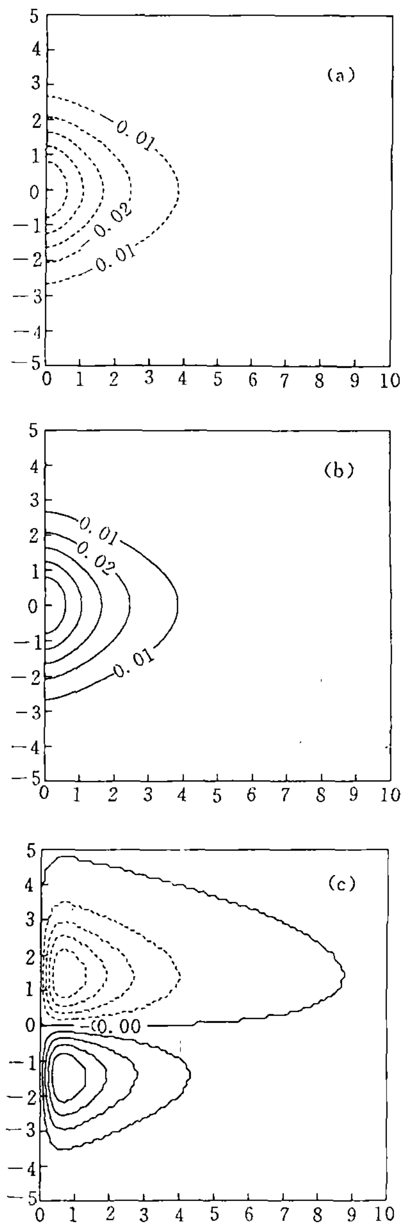


Fig. 8. As in Fig. 7 but $\alpha_1 = \alpha_2 = 4.0$. The interval is 0.0005 in diagram c.

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