DESIGNATION AND APPLICATION OF A SPHERICAL LONG-WAVE SPECTRAL MODEL

Wu Guoxiong (吴国雄)

Institute of Atmospheric Physics, Academia Sinica, Beijing

Received August 15, 1988

ABSTRACT

A two-level, quasi-geostrophic long-wave model based on spherical coordinates was developed with the explicit part belonging to a low-order model. However, it includes not only diabatic heating, Ekman friction and mountain distribution, but also parameterized forcing effects of transfer properties of transient eddies.

Experiment results showed that, due to the introduction of the parameterization of transfer properties of transient eddies, remarkable improvements on characters of low-order model had been obtained. In addition to its economization in calculation and conciseness in physics as in a low-order model, the long-wave model was shown to describe the energetics and angular momentum balance of the atmosphere much more reasonably, and to present the features of zonal mean westerlies and stationary waves much more correctly than the corresponding low-order model. This kind of long-wave model was therefore regarded as suitable for theoretical research and numerical modelling of some aspects of the general circulation of the atmosphere.

I. INTRODUCTION

Numerical weather prediction (NWP) emphasizes on simulating the real evolutions of atmospheric circulation. Information obtained from NWP based on precise initial fields and on sophisticated dynamical and physical frameworks is almost as complicated as those observed in the real atmosphere, and can hardly help us in understanding causes why atmospheric motions evolve. Comparison experiments based on general circulation modelling (GCM) can give some perspective mechanism on the atmospheric motions. However, the quadratic relation between transferring and transferred quantities, the nonlinear interaction between dynamic forcing and thermal forcing, and the feedback between inner variables and external parameters make the physical pictures of mechanism blurred. Dynamic investigation requires another category of model which is able to reveal cause and result conveniently.

Platzman (1960) proposed the concept of low-order model (LOM), and Lorenz (1960) obtained the maximum simplification principle for the equation system of atmospheric motions. By only specifying a few modes, this equation system can be reduced to a limited number of tendency equaton of spectral coefficient. Therefore, LOM is economic in calculation and concise in physics. During the last decade, the application of LOM has greatly accelerated the development of nonlinear dynamics.

However, it is readily shown that LOM cannot correctly describe the balance of atmospheric angular momentum and the maintenance of zonal mean westerlies. In the atmosphere, the direct energy conversion between zonal mean potential energy and kinetic energy ($\overline{K}$) is very weak. Frictional dissipation reduces $\overline{K}$ constantly. Part of $\overline{K}$ is converted to wave energy due to the work done by negative mountain torque. The development of planetary waves during upward propagation also abstracts energy from $\overline{K}$. Therefore, zonal mean westerlies must be maintained by other kind of mechanism. Stationary waves with synoptic scales can supply energy to zonal flow. However, angular momentum transfer by even total stationary waves is only one third of that by transient eddies (Oort and Rasmusson, 1971). Therefore, the maintenance of zonal kinetic energy depends mainly upon the energy conversion by transient eddies. Such mechanism is neglected in LOM and replaced by a given forcing source.

Based upon the theory on parameterization of transfer properties of baroclinic transient eddies developed by Green (1970), White and Green (1982) proposed a β-plane long wave model. Since β-effect is important in the dynamics of global general circulation, it should be included into the model in order to tackle global problems. Considering the constraint of atmospheric angular momentum balance, Wu (1983) developed a long wave spectral model (LWM) based on spherical coordinates. The most outstanding feature of this long wave model is that its explicit framework is basically that of a corresponding low-order model (LOM), while its implicit part represents the effects of transfer properties of transient eddies. The introduction of the effects of constraint of angular momentum balance and the mechanism for the maintenance of westerlies makes LWM not only possess the advantages of LOM such as economy and conciseness, but also overcome to a large extent the disadvantages of LOM as discussed at the beginning. After a LWM is designed in Section II, its energetics is analysed in Section III. In Section IV, comparisons of results from the LWM with those from observations and general circulation models are given. Comparisons between LWM and LOM are presented in Section V. Finally, the possibility for further development of LWM is discussed in Section VI.

II. BASIC EQUATION SYSTEM

In order to correctly describe the balance of atmospheric angular momentum, a model atmosphere should have the ability to reasonably present the β-effect and the zonal distribution of Richardson Number. One way to achieve this is to build a long wave model based on spherical coordinates rather than β-plane approximation. In doing so, the transfer coefficient κ of baroclinic waves must be carefully dealt with (see Wu and White, 1986).

1. *Model Development*

In quasi-geostrophic framework, vorticity and thermodynamic equations can be expressed respectively as

$$\frac{D}{Dt}(\zeta+f)=f_0\frac{\partial w}{\partial z}-\nabla\cdot(V'\zeta'), \tag{1}$$

$$\frac{D}{Dt}\left(\frac{\partial\psi}{\partial z}\right)+\frac{N^2}{f_0}w=-r\left(\frac{\partial\psi}{\partial z}-\frac{\partial\tilde{\psi}}{\partial z}\right)-\nabla\cdot\left(V'\frac{\partial\psi'}{\partial z}\right), \tag{2}$$

where the term involving primes denotes the covergence of transient eddy flux, representing the forcing effect of transient eddies on zonal mean quantities and stationary waves. Boussinesq approximation is adopted here since it simplifies problem and will not distort mountain torque in angular momentum balance (Wu, 1984). Instead of prescribed heating, the Newtonian heating is employed in the thermodynamic equation (2) since in the real atmosphere, the distribution of heating source and sink is not only the cause, but also the result, of the general circulation of the atmosphere. The reason why we can deal with stationary and transient eddies separately lies in the fact that the wavelength of the most unstable baroclinic wave is at least twice smaller than that of resonant waves. In addition, the excitation and behavior of these two kinds of eddies are quite different. Stationary waves are generated mainly by external forcing and amplified during upward propagation towards the stratosphere, whereas synoptic scale waves are excited mainly by baroclinic instability and evanescent in the vertical.

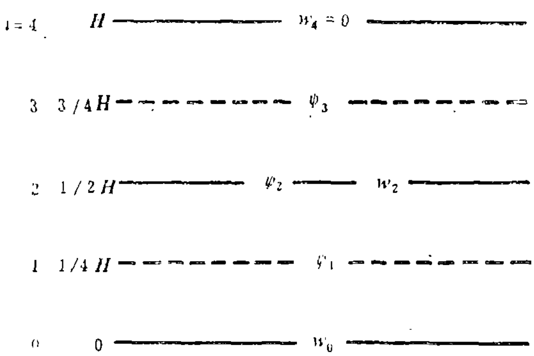

Fig. 1. Vertical resolution of the model atmosphere. Mechanical forcing is exerted at the surface; thermal forcing, at the interface between the two layers; and vorticity forcing, at levels 1 and 3.

The model atmosphere is divided into two layers in the vertical with equal depth (see Fig. 1). The upper boundary at $z=H=10$ km is taken as a rigid lid. This may reduce poleward heat transfer (Shutts, 1978), but will not affect the problems to be treated below. Eqs. (1) and (2) are written respectively at odd levels and the interface. Vertical motion at lower boundary w_0 is considered to result from orography and Ekman pumping and suction, i. e.

$$w_0 = \boldsymbol{V}_0\cdot\nabla h+\alpha\nabla^2\psi_0, \tag{3}$$

where $\alpha=(\kappa/2f_0)^{1/2}$, κ is eddy viscosity coefficient, and h is mountain height. Quantities at lower boundary with subscript "o" are extrapolated from those at levels 1 and 3. After laborious manipulations and non-dimensionalization, we can obtain

$$\left\{\begin{aligned} \nabla^2\frac{\partial\psi_M}{\partial t} &= -\frac{1}{2}[J(\psi_M,\nabla^2\psi_M)+J(\psi_s,\nabla^2\psi_s)]-J(\psi_M-\psi_s,\eta)\\ &\quad -2\frac{\partial\psi_M}{\partial\lambda}-R_d(\nabla^2\psi_M-2\nabla^2\psi_s)-\nabla\cdot(\boldsymbol{V}_1'q_1'+\boldsymbol{V}_3'q_3'),\\ (\nabla^2-\mu^2)\frac{\partial\psi_s}{\partial t} &= -\frac{1}{2}[J(\psi_M,(\nabla^2-\mu^2)\psi_s)+J(\psi_s,\nabla^2\psi_M)]+J(\psi_M-\psi_s,\eta)\\ &\quad -2\frac{\partial\psi_s}{\partial\lambda}+R_d(\nabla^2\psi_M-2\nabla^2\psi_s)+\mu^2\gamma^*(\psi_s-\tilde{\psi}_s)-\nabla\cdot(\boldsymbol{V}_3'q_3'-\boldsymbol{V}_1'q_1'), \end{aligned}\right. \tag{4}$$

where $\psi_s=\psi_3-\psi_1$, $\psi_M=\psi_3+\psi_1$, quasi-geostrophic potential vorticity

$$\left\{\begin{aligned} q_3 &= \zeta_3+f-\frac{\mu^2}{2}\psi_s,\\ q_1 &= \zeta_1+f+\frac{\mu^2}{2}\psi_s, \end{aligned}\right. \tag{5}$$

non-dimensional coefficient

$$\mu^2=\frac{8f_0^2a^2}{N^2H^2},\quad R_d=\frac{f_0a}{\Omega H},\quad r^*=\frac{r}{\Omega},\quad \eta=\frac{f_0h}{\Omega H},\quad f=2\sin\varphi=2x,$$

and non-dimensional Laplacian and Jacobin operator are respectively

$$\boldsymbol{V}^2=a^2\nabla^2,\quad \boldsymbol{J}(A,B)=a^2J(A,B).$$

Mountain itself does not generate or dissipate total kinetic energy. Therefore, the wind employed in the mountain terms in Eq. (4) is at level 1 rather than at the surface so that the consistency of the difference scheme is obtained.

2. *Model Solution*

In an isolated frictionless system with uniform surface, the quasi-geostrophic potential vorticity q is conserved, its eddy flux can then be parameterized by using Green's (1970) theory. We then have

$$V'_i q'_i=-\kappa_i^*\boldsymbol{V}q_i,(i=1,3),\tag{6}$$

where non-dimensional transfer coefficient $\kappa_i^*=a^{-2}\Omega\kappa_i(\varphi)$, which reaches the maximum at 35°N and vanishes at the equator and poles. It is readily shown that such parameterization results in direct dissipative effect of transient eddies on stationary waves. However, as we will see later, the indirect effect of transient eddies via mean flow on the intensification of stationary waves is more important. Substituting Eq. (6) into (4), we obtain

$$\begin{pmatrix}\boldsymbol{V}^2\dfrac{\partial\psi_M}{\partial t}\\ (\boldsymbol{V}^2-\mu^2)\dfrac{\partial\psi_s}{\partial t}\end{pmatrix}=\begin{pmatrix}G_1\\G_3\end{pmatrix}(\psi_M,\psi_s;\eta,\tilde{\psi}_s;\kappa_1^*,\kappa_3^*).\tag{7}$$

The functional forms of G_1 and G_3 are referred to Wu (1984). Upon choosing spherical spectral function as base function, the variables ψ_M, ψ_s and forcing functions $\tilde{\psi}_s$ and η can be expanded as

$$\begin{pmatrix}\psi_M\\ \psi_s\\ \tilde{\psi}_s\\ \eta\end{pmatrix}=\sum_n\sum_m\left\{\begin{pmatrix}A_n^m(t)\\ a_n^m(t)\\ SA_n^m(t)\\ hA_n^m(t)\end{pmatrix}\cos m\lambda+\begin{pmatrix}B_n^m(t)\\ b_n^m(t)\\ SB_n^m(t)\\ bB_n^m(t)\end{pmatrix}\sin m\lambda\right\}P_n^m(x),\tag{8}$$

$$\kappa_i^*=k_iP_3^1(x)\quad(i=1,3).\tag{9}$$

The constant k_1 correspondes to $\kappa_1=4\times10^6\mathrm{m^2s^{-1}}$, $k_3=pk_1$, and the value of p is determined by the constraint that the sum of frictional torque $\overline{T}_f^\lambda$ and mountain torque $\overline{T}_M^\lambda$ over the whole hemisphere is zero since in a long period, atmospheric angular momentum is conserved, and no net generation is required, i. e.,

$$\int_0^1(\overline{T}_f^\lambda+\overline{T}_M^\lambda)dx=0.\tag{10}$$

$P_n^m(x)$ in formulas (8) and (9) is the normalized associated Legendre function determined by the Rodrigues formula

$$P_n^m(x)=\left[\frac{(2n+1)(n-m)!}{2(n+m)!}\right]^{1/2}\frac{(1-x^2)^{m/2}}{2^nn!}\frac{d^{(n+m)}}{dx^{n+m}}(x^2-1)^n.\tag{11}$$

In order to trace the connection between physics and the model results as clearly as possible, we try to avoid dealing with a complicated model. According to the maximum simplification principle (Lorenz, 1960), we choose two modes P_2^0 and P_4^0 for the zonal mean, and four modes P_2^1, P_4^1, P_3^2 and P_5^2 for the zonally asymmetric part. From (7) to (11), we may obtain 20 prognostic equations

$$\begin{pmatrix} \dot{A}_n^m(t) \\ \dot{B}_n^m(t) \\ \dot{a}_n^m(t) \\ \dot{b}_n^m(t) \end{pmatrix} = \begin{pmatrix} \mathbf{F}_1 \\ \mathbf{F}_2 \\ \mathbf{F}_3 \\ \mathbf{F}_4 \end{pmatrix} (A_n^m, B_n^m, a_n^m, b_n^m; SA_n^m, SB_n^m, hA_n^m, hB_n^m; p), \qquad (12)$$

and one constraint on angular momentum balance

$$p = (-d_1/d_3) + d_s. \qquad (13)$$

The functional forms of $\mathbf{F}_i\,(i=1\text{—}4), d_1, d_3$ and d_s are referred to Wu (1984). System (12) and (13) contain 21 equations and 21 variables, and can be solved numerically. In order to guarrantee computational stability, time forward difference is applied to frictional term and the term representing transient eddy effects (see Wu, 1984). The temporal increment is taken as 6 hours. All integrations are started from a resting atmosphere and carried to a quasi-steady state until the sequential integrated values of p at the $(j+1)$th time step satisfy the accuracy requirement

$$|p^{j+1} - p^j| < 10^{-5}. \qquad (14)$$

It has been shown that such quasi-steady state of wave motion is consistent with theoretical solutions of stationary waves provided that the zonal flow is the same (Wu, 1986). Results from this state, referred to as the model atmosphere, are diagnosed in Sections IV and V.

III. ENERGETICS OF THE LONG WAVE MODEL

Let the total kinetic energy K and potential energy A respectively be

$$\begin{cases} K = \dfrac{1}{4}\displaystyle\int_M (|\Delta\psi_m|^2 + |\nabla\psi_s|^2)\,dm, \\ A = \dfrac{\nu^2}{2}\displaystyle\int_M \psi_s^2\,dm. \end{cases} \qquad (15)$$

Separating these into parts of zonal mean $(\bar{K}, \bar{A})$ and perturbation (K^*, A^*), then after some calculations, one can obtain the following energy equations:

$$\begin{cases} \dfrac{d\bar{A}}{dt} = -C(\bar{A}-A^*) - C(\bar{A}-\bar{K}) - C_t(\bar{A}-\bar{K}) + \bar{G}, \\ \dfrac{dA^*}{dt} = C(\bar{A}-A^*) - C(A^*-K^*) - C_t(A^*-K^*) + G^*, \\ \dfrac{d\bar{K}}{dt} = C(K^*-\bar{K}) + C(\bar{A}-\bar{K}) + C_t(\bar{A}-\bar{K}) - C_M(\bar{K}-K^*) - \bar{D} - \bar{D}_t, \\ \dfrac{dK^*}{dt} = -C(K^*-\bar{K}) + C(A^*-K^*) + C_t(A^*-K^*) + C_M(\bar{K}-K^*) - D^* - D_t^*, \end{cases} \qquad (16)$$

where, $C(x-y)$ represents the energy conversion from the form of x to that of y; G, generation; D, dissipation; subscript t denotes contributions from transient eddies. $C_M(\bar{K} - K^*)$ is the kinetic energy conversion due to orography, and is equivalent to the "primary source" defined by Eliassen and Palm (1960). Other terms in the equation possess the general meaning as in ordinary studies of atmospheric energetics by, for example, Oort (1964), Oort and Peixoto (1974), Peixoto and Oort (1974), Holton (1979), and Yao (1980).

1. *Direct Dynamic Effect of Orography*

In Eq. (16), $C_M(\bar{K}-K^*)=-\int_S \rho_0 f_0 \bar{u}_0 \overline{v_0^* h^*} ds$, where S is the hemispheric area. Notice that mountain torque $\bar{\tau}_m = f_0 \rho_0 \overline{v_0^* h^*}$ then

$$C_M(\bar{K}-K^*)=-\int_S \bar{u}_0 \bar{\tau}_m ds. \tag{17}$$

At the surface, vertical wave energy flux (or, EP-flux) can be written as

$$\overline{P_0^* w_0^*}=\bar{u}_0 \overline{P_0^* \frac{\partial h^*}{\partial x}}=-\bar{u}_0 \bar{\tau}_m. \tag{18}$$

Therefore

$$C_M(\bar{K}-K^*)=\int_S \overline{p_0^* w_0^*} ds. \tag{19}$$

Equations (17)—(19) show that, the work done by negative mountain stress upon basic flow equals just the upward wave energy flux, and their area integral over the hemisphere amounts to the energy conversion from zonal mean kinetic energy to perturbation kinetic energy. Such orography effect will not alter total kinetic energy as evident in Eq. (16). However, this mountain stress can reduce the westerly angular momentum $\bar{\mu}=\int_0^H \rho \bar{u} a \cos\varphi dz$, since over the whole hemisphere S

$$\frac{d}{dt}\int_S \bar{\mu} ds=\int_S \bar{\tau}_m a \cos\varphi ds + \text{other terms}, \tag{20}$$

and since $\bar{\tau}_m<0$ (refer to Wu, 1984). This is simply because perturbation velocity does not contribute to total angular momentum. Therefore, when westerlies are changed to perturbation winds as impinging upon orography, although their total kinetic energy is conserved, their total angular momentum experiences a net loss.

2. *Effect of Transient Eddies*

The LWM energetics expressed by Eq. (16) can be exhibited by Fig. 2b against that of a complete GCM shown as Fig. 2a. Compared with LOM, the energetics in LWM includes additional terms, i. e. dissipation terms $\bar{D}_t, D_t^*$ and conversion terms $C_t(\bar{A}-\bar{K})$ and $C_t(A^*-K^*)$. The latter two terms, i. e.

$$\begin{cases} C_t(\bar{A}-\bar{K})=-\dfrac{g}{B}\displaystyle\int_M \overline{\overline{v_2'\delta\phi'}\dfrac{\partial \delta\phi}{\partial y}} dm \\ C_t(A^*-K^*)=-\dfrac{g}{B}\displaystyle\int_M (\overline{V_2'\delta\phi'})^* \cdot \nabla\delta\phi^* dm \end{cases} \tag{21}$$

are functions of transfer coefficient κ. Obviously, the transient eddy transfer of transient enthalpy down the gradient of zonal mean (stationary perturbation) enthalpy results in the release of zonal mean (stationary perturbation) enthalpy and then maintains the zonal mean (stationary perturbation) kinetic energy. These behave in the same way as in the real atmosphere (Lau, 1979), but are excluded in LOM. There is no doubt that the energetics of LWM is much more reasonable than that of LOM.

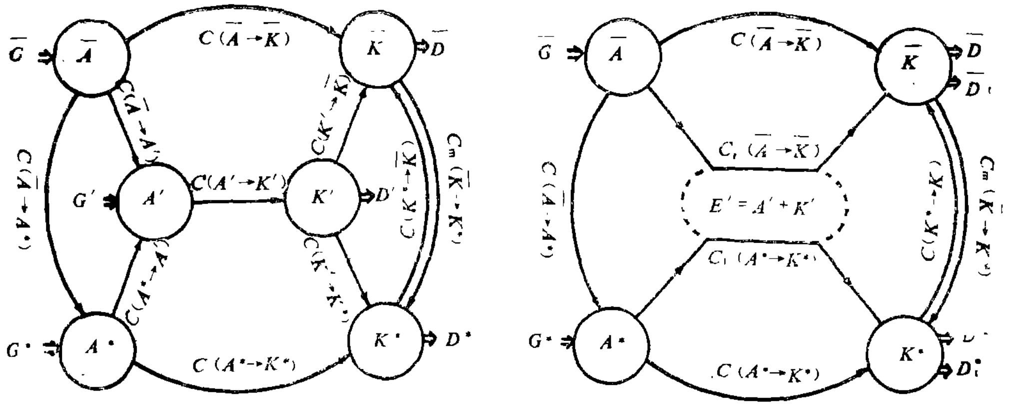

Fig. 2. The energetics of the atmosphere, —, $*$, and ' denote respectively quantities associated with zonal mean, stationary waves and transient eddies. (a) General circulation model; (b) long-wave model.

A complete GCM energetics (see Fig. 2a) also contains potential energy A' and kinetic energy K' of transient eddies, and the associated energy conversions. Firstly, we notice that the mathematical expressions of the terms $\bar{G}, G^*, \bar{D}, D^*, C_M(\bar{K}-K^*), C(\bar{A}-A^*)$, $C(\bar{A}-\bar{K})$, $C(A^*-K^*)$ and $C(\bar{K}-K^*)$ in GCM are respectively identical with those in LWM. In addition, the expressions of $C(\bar{A}-A')$ and $C(A^*-A')$ in GCM are the same as $C_t(\bar{A}-\bar{K})$ and $C_t(A^*-K^*)$ in LWM respectively. Therefore, energy balance for $\bar{A}$ and A^* in these two models, i.e. GCM and LWM, is mathematically identical. Secondly, from the point of view of slantwise instability, when a potentially warm parcel moves upwards and northwards along a trajectory with its slop smaller than that of isentropic surface, sensible heat is transferred upwards and northwards simultaneously. Thus, $C(\bar{A}-A')$ and $C(A'-K')$ happen almost simultaneously, and so do $C(A^*-A')$ and $C(A'-K')$. These make it feasible to combine A' and K' as a unity of total transient eddy energy E'. Therefore, from the standpoint of energy budget, the behavior of zonal mean flow and stationary waves in LWM and GCM could be respectively identical provided that the following conditions are satisfied:

$$\begin{cases} C(K'-\bar{K})_{GCM}=[C_t(\bar{A}-\bar{K})-\bar{D}_t]_{LWM}, \\ C(K'-K^*)_{GCM}=[C_t(A^*-K^*)-D_t^*]_{LWM}. \end{cases} \tag{22}$$

The right hand sides of (22) are all functions of transfer coefficient κ. Therefore, the justification of (22) depends on the choice of κ. In other words, the reasonable choice of κ is the key to the design of a long wave model. For the sensitivity of LWM to the choice of κ, readers are referred to Wu and White (1986).

IV. COMPARISON BETWEEN RESULTS FROM LWM AND FROM GCM AND OBSERVATIONS

1. *No Mountain Model (NOM)*

In Eq. (12), let

$$\begin{cases} hA_n^m=hB_n^m=0; \\ SA_2^0=-22\mathrm{K}/T_s, SA_4^0=3\mathrm{K}/T_s, \\ SA_2^1=-10\mathrm{K}/T_s, SB_2^1=-15\mathrm{K}/T_s, \\ SA_3^2=37\mathrm{K}/T_s, SA_5^2=5\mathrm{K}/T_s, \end{cases} \tag{23}$$

where $T_s=2f_0\Omega a^2R^{-1}=1750\mathrm{K}$ is a constant temperature measure. At steady state, the distribution of diabatic heating of the model atmosphere determined by (23) resembles the average state of northern hemispheric winter.

Fig. 3 shows the latitudinal distributions of surface frictional torque in LWM atmosphere in comparison with analyses. Results of Priestly (1951) and Kung (1968) are adopted since a drag coefficient $C_d=0.0016$ over oceans was employed in their calculations so that the results are logically more comparable with those from no mountain model. Based on sea surface data, Hellerman (1968) obtained similar results as Priestly's. Fig. 3 shows that the intensity as well as distribution of frictional torque in the LWM are very similar to those obtained from observations. The latitudinal distributions of eddy angular momentum flux are shown in Fig. 4. This transfer in LWM is weaker. In addition, since polar easterlies can not exist in a model with only zonal modes P_2^0 and P_4^0, the equatorward flux in high latitudes disappears, and the mid latitude maximum is shifted northwards by about 5 degrees. These can be solved by introducing new base modes such as P_6^0. In view of the high truncation of the model, results from Figs. 3 and 4 are indeed encouraging.

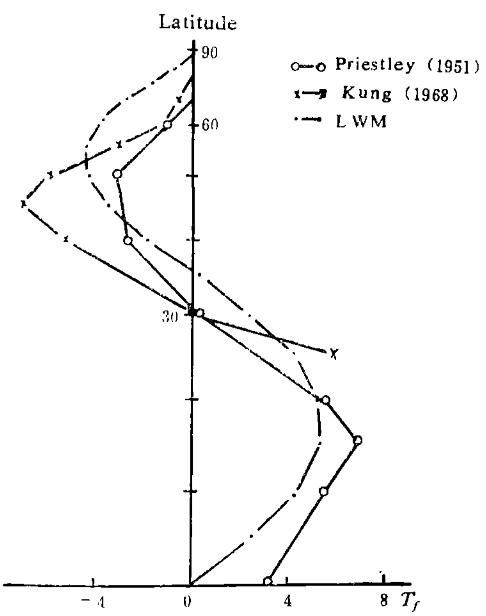

Fig. 3. The mean zonal eastward torque per 5 latitude belt exerted upon the atmosphere by surface friction in the LWM case and in the real atmosphere. Units in Hadley (10^{18} kg m^2 s^{-2}).

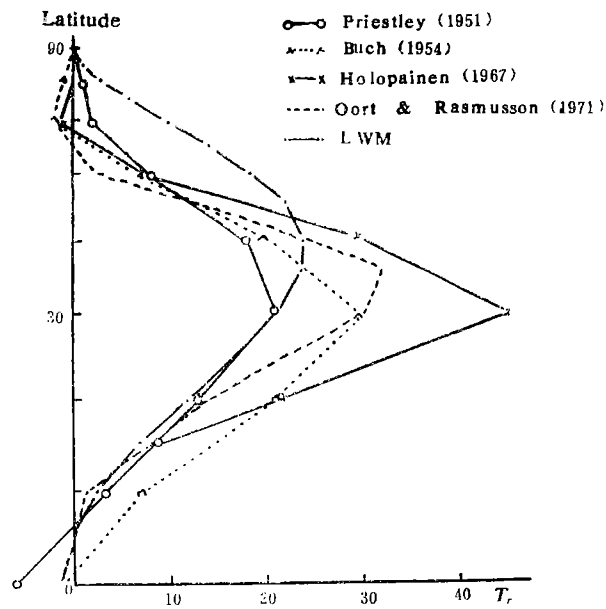

Fig. 4. The poleward transfer of westerly angular momentum by eddies in the LWM case and in the real atmosphere. The estimate by Priestley comes from the requirement of balanc of angular momentum. Units in Hadley (10^{18}kg m^2 s^{-2}).

2. *Mountain Model (MTN)*

Upon introducing the M-forcing given by

$$hA_2^0=-100\text{m}/H_s,$$
$$hA_2^1=-30\text{m}/H_s,\quad hB_2^1=20\text{m}/H_s,$$
$$hA_4^1=-100\text{m}/H_s,\quad hB_4^1=-500\text{m}/H_s,$$

$$hA_3^2=150\mathrm{m}/H_s,\quad hB_3^2=-50\mathrm{m}/H_s,$$

into Eq. (23), the main effects of Qinghai-Xizang Plateau and Rockies on atmospheric angular momentum balance are captured, and the mountain model (MTN) is formed. Here, $H_s=\Omega H f_0^{-1}=7854$ m is a constant height measure. The latitudinal distributions of mountain torque T_M and frictional torque T_f are shown in Fig. 5 in comparison with their counterparts in the atmosphere as obtained by Newton (1971). They all show that over most latitudes, T_M and T_f possess the same sign, i. e. both positive in low latitudes and negative in mid latitudes.

Another important quantity correlated to surface torque is the mean meridional circulation χ. From the zonal mean continuity equation, χ can be defined as

$$\begin{cases} 2\pi a^2\cos\varphi\,\rho\bar{w}=-\dfrac{\partial\chi}{\partial\varphi}, \\ 2\pi a\cos\varphi\,\rho\bar{v}=\dfrac{\partial\chi}{\partial z}, \end{cases} \tag{24}$$

negative χ then corresponds to direct circulation. The intensity of the Ferrel circulation in the LWM is about half of the real atmosphere (Palmen and Vuorela, 1963). However, the Hadley cell is too weak. This is firstly because the LWM used here is a quasi-geostrophic model. In such a model, coefficient $\left(f-\dfrac{\partial\bar{u}}{\partial y}\right)$ is replaced by a constant f_0 with the mid latitude value of f, so that the f-effect in low latitude is exaggerated. Since the vertical westerly shear required for balancing the zonal mean temperature gradient is proportional to $\left(f-\dfrac{\partial\bar{u}}{\partial y}\right)(\bar{v}_3-\bar{v}_1)$, for a given temperature gradient, a larger f will render a smaller mean meridional speed. This can also be observed in other geostrophic

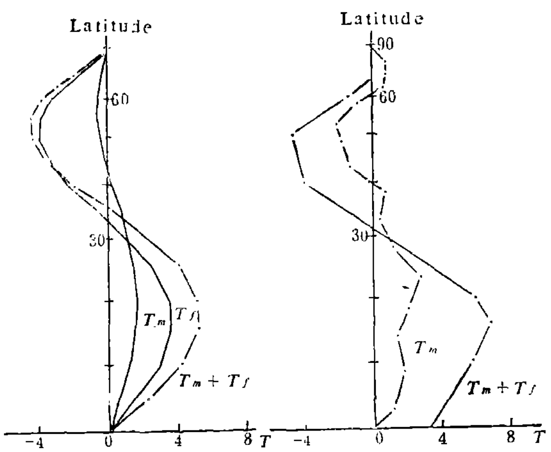

Fig. 5. The distribution of mountain torque T_M, frictional torque T_f, and the total surface torque T_M+T_f in the LWM (left) and the T_M(Newton, 1971) and T_M+T_f (Priestley, 1951, modified data) in the real atmosphere (right).Units in Hadley ($10^{18}\mathrm{km}^2\ \mathrm{s}^{-2}$).

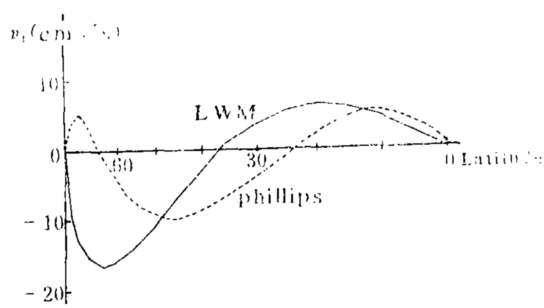

Fig. 6. The zonal mean meridional velocity in the upper layer ($\bar{v}_3$) of the long wave model and of the Phillips' (1954) model based on the β-plane. The mid-point and boundary ($-L$ and L) values in the β-channel have been taken to lie at 45° N, 0 and 90° N for the convenience of comparison.

models. Using a two-layer quasi-geostrophic GCM with β-plane approximation, Phillips (1954) obtained even smaller $\bar{v}^3$ than that in LWM (see Fig. 6). Using a similar model to Phillips' but with spherical coordinates, Hollingsworth (1975) obtained the results very similar to ours. Another maybe more important reason for the weakness of the Hadley cell in LWM is that the model does not include latent heating. GCM experiments of Manabe and Smagorinsky (1967) show that the annual mean intensity of the Hadley cell is about 140×10^6 ton s^{-1} in a moist model, but only 52×10^6 ton s^{-1} if there is no latent heat release.

V. COMPARISON BETWEEN RESULTS FROM LWM AND LOM

Letting $\kappa_i\equiv0\,(i=1,3)$ in Eq. (7) or (12), the LWM then retrogrades to LOM. The influence of transient eddies on atmospheric motions can be shown by comparing results from LWM with those from LOM. For simplicity, consider only zonal symmetric heating. Then, external forcing is only from orography

$$hA_3^2=-2000\text{m}/H_s,\ hB_2^1=1000\text{m}/H_s. \tag{25}$$

At steady states, the zonal mean wind profiles at different levels are shown in Fig. 7. In LOM, surface wind becomes easterly, and Edelmann phenomena appear. Due to the introduction of effects of transient eddy flux of angular momentum, this is overcome in LWM. The parameterization of transfer properties of transient eddies results in more energy conversion from zonal mean potential to kinetic energy, the maximum westerly at upper level in LWM is twice as large as that in LOM.

Fig. 8 shows the distributions of upper level stationary waves at 40° N in the two models. Its amplitude in LWM is about 6 times as large as in LOM. In order to analyse the cause, we assume that: 1) the intensification of wave amplitude at 40° N represents the hemispheric features; and 2) this intensification is proportional to the energy conversion rate towards wave perturbation. Then, we can ignore the convergence of wave energy flux in the north-south direction, and pay attention only to the energy convergence per unit area within unit width along 40°N, i. e.

$$C_{40^\circ\text{N}}(\bar{E}-E^*)=\bar{\rho}^2H\left[\frac{f_0}{B\bar{T}}\frac{\partial\bar{u}}{\partial z}\overline{(v^*T^*)}-\frac{\partial\bar{u}}{\partial y}\overline{(v^*u^*)}\right]_{40^\circ\text{N}}. \tag{26}$$

Each term on the right hand side of Eq. (26) for the two models is calculated and shown in Table 1. Obviously, the vertical and horizontal wind shear, and the horizontal flux of momentum and sensible heat in LWM are all increased more prominently than in LOM. Thus, energy conversion in LWM is about 5.5 times as large as in LOM. Although the direct orography effect in LWM is also increased, it is much smaller than the "secondary" source as described by Eq. (26) and defined by Eliassen and Palm (1960). It turns out to be clear that the great intensification of stationary waves in LWM is due to the introduction of effects of baroclinic transient eddies so that the basic westerlies and the transfer properties of the atmosphere are presented better, and more energy are transferred from basic flow to stationary waves.

In pure mechanical forcing case, more dramatic changes occur in the surface pressure field (Fig. 9). Despite the fact that high pressure ridge all located on the western side of mountain, in LOM, air flows westwards and Edelmann phenomena appear. Whereas in LWM, there appear closed high and low systems, and westerlies exist in mid and high latitudes. This remarkable improvement is due to the introduction of poleward vorticity flux of baroclinic transient eddies so that angular momentum balance is presented correctly (Wu, 1983; 1984).

Table 1. Various Zonal Mean Quantities at 40° N in the Two Models

Model	$\partial\bar{u}/\partial z$ $(2\times10^{-4}s^{-1})$	$\overline{v^*T^*}$ $(K\ m\ s^{-1})$	$\partial\bar{u}/\partial y$ $(10^{-6}s^{-1})$	$\overline{v^*u^*}$ (m^2s^{-2})	$C_{40°}(\bar{E}-E^*)$ (Wm^{-2})
LWM	12	0.88	3.1	3.78	0.55
LOM	8	0.18	−0.5	0.07	0.10

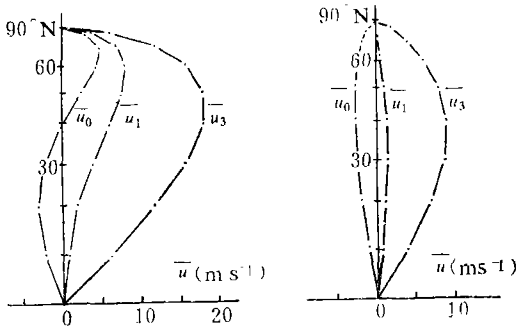

Fig. 7. Distribution with latitude of the zonal mean wind at different levels. (a) In long wave model (left); (b) in low order model (right).

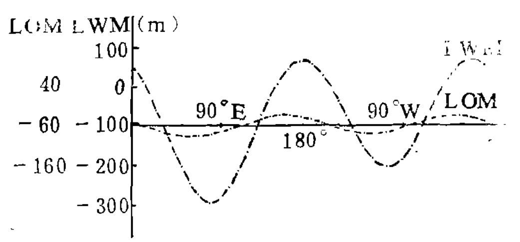

Fig. 8. Mechanically forced stationary waves in the upper layer of the model atmosphere in long wave model (LWM) and in low order model (LOM).

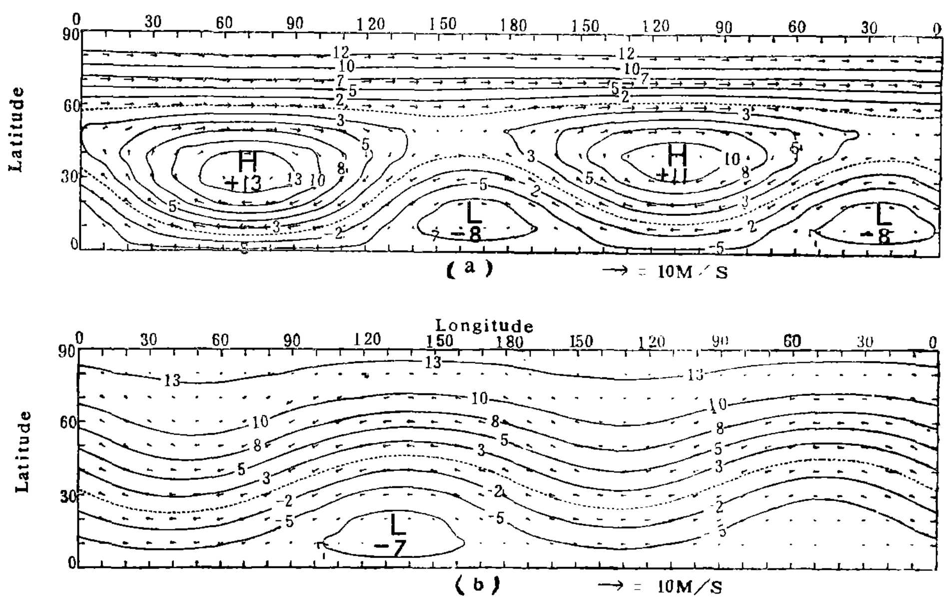

Fig. 9. The response of the surface pressure field (units in hPa) to mechanical forcing. The orography used is given by $hA_3^2=-2000\text{m}/H_s, hB_2^1=1000\text{m}/H_s$. (a) Long wave model; (b) low order model.

VI. DISCUSSIONS AND CONCLUSIONS

(1) Since the theory on LOM was proposed by Platzman and Lorenz in 1960, it has been widely used due to the fact that it is economy, concise and easy for tracing nonlinear

behavior of atmospheric motions and describing some important atmospheric phenomena. The application of LOM has also catalyzed the development of dynamic meteorology. There is no doubt that LOM will still be widely used and continue to develop.

However, LOM possesses only a few freedoms and is limited in application. Firstly, as warned by Lorenz at the beginning, wave shape depends upon phase critically. It is therefore difficult to use LOM to present wave motions with their shape unchanged. More importantly, high truncation distorts the maintenance of zonal westerlies and the mechanism of angular momentum balance. These result in the weakening of the transfer properties, the intensity of zonal westerlies and the amplitude of stationary waves, of the model atmosphere. In the case of pure mechanical forcing, unrealistic equilibrium in pressure field at the surface may occur. Therefore, when LOM is employed to tackle these problems, care is needed. It is always helpful to compare its results with those from GCM or from observations.

(2) In the long wave model developed here, the transfer properties of baroclinic transient eddies are parameterized. Since this is a very important mechanism for the maintenance of westerlies and angular momentum balance, then in LWM, the disadvantages of LOM mentioned above are overcome, the atmospheric energetics and angular momentum balance are presented more reasonably, and the main characters of atmospheric stationary waves are captured. Since the explicit part of the LWM is, in fact, the frame of corresponding LOM, LWM then remains the advantages of LOM such as concise in mechanism and economical in calculation. For a computer in the order of millions per second, one-year integration requires only a few minutes in calculation.

(3) The parameterized part of the LWM depends critically on the choice of transfer coefficient κ. Wu adopted $P_3^2(x)$ to present the latitudinal variation of κ. Results seem to be satisfactory. However, the eddy transfer of heat and momentum is somewhat weaker. This may be due to the choice of upper rigid boundary and the weakness of stationary waves. It may also be due to that the choice of κ is not accurate. Wiin-Nielsen and Sela (1971) have evaluated the transfer coefficient κ of quasi-geostrophic potential vorticity, it possesses minimum at two extreme latitudes and a pulse-like maximum in mid latitudes. Their evaluation is valuable for the further improvement of LWM. On the other hand, as in LOM, LWM possesses small freedom, it would be unrealistic to use LWM to present wave behavior without changing its shape. However, experiments (see Wu, 1984) have shown that LWM is capable and successful in describing some important dynamic features of stationary waves.

REFERENCES

Eliassen, A., and Palm E. (1960), On the transfer of energy in stationary mountain waves, *J. Geof. Publ.*, **22:** 3, 1—23.

Green, J.S.A. (1970), Transfer properties of the large-scale eddies and the general circulation of the atmosphere, *Quart. J. Roy Meteor. Soc.*, **96:** 157—185.

Hellerman, S. (1967), An updated estimate of the wind stress on the world ocean, *Mon. Wea. Rev.*, **95:** 607—626.

Hollingsworth, A. (1975), Baroclinic instability of a simple flow on the sphere, *Quart. J. Roy. Meteor. Soc.*, **101:** 495—528.

Holton, J.R. (1979), *An Introduction to Dynamic Meteorology*, Academic Press.

Kung, E.C. (1968), On the momentum exchange between the atmosphere and earth over the Northern Hemisphere, *Mon. Wea. Rev.*, **96:** 337—341.

Lau, N.C. (1979), The observed structure of tropospheric stationary waves and the local balance of vorticity and heat, *J. Atmos. Sci.*, **36**: 996—1016.

Lorenz, E.N. (1960), Maximum simplification of the dynamic equations, *Tellus*, **12**: 243—254.

Lorenz, E.N. (1982), Low-order models of atmospheric circulations, *J. Meteor. Soc. Japan*, **60**: (1), 255—267.

Manabe, S., and Smagorinsky J. (1967), Simulated climatology of a general circulation model with a hydrologic cycle (II. Analysis of the tropical atmosphere), *Mon. Wea. Rev.*. **95**: 155—169.

Newton, C.W. (1971), Mountain torques in the global angular momentum balance, *J. Atmos. Sci.*, **28**: 623—628.

Oort, A.H. (1964), On estimates of the atmospheric energy cycle, *Mon. Wea. Rev.*, **92**: 483—493.

Oort, A.H., and Peixoto J.P. (1974), The annual cycle of the energetics of the atmosphere on a planetary scale, *J. Geophy. Res.*, **79**: 2705—2719.

Oort, A.H., and Rasmusson E.M. (1971), Atmospheric circulation statistics, NOAA, professional paper 5.

Palmen, E., and Vuorela L. (1963), On the mean meridional circulation in the Northern Hemisphere during the winter season, *Quart. J. Roy. Meteor. Soc.*, **89**:131—138.

Peixoto, J.P., and Oort A.H. (1974), The annual distribution of atmospheric energy on a planetary scale, *J. Geophy. Res.*, **79**: 2149—2159.

Phillips, N.A. (1954), Energy transformations and meridional circulations associated with simple baroclinic waves in a two-level quasi-geostrophic model, *Tellus*, **6**: 273—286.

Platzman, G.W. (1960), The spectral form of the vorticity equation, *J. Meteor.*, **17**: 635—644.

Priestley, C.H.B. (1951), A survey of the stress between the ocean and atmosphere. *Australian J. Sci. Res.*, 315—328.

Shutts, G.J. (1978), Quasi-geostrophic planetary wave forcing, *Quart. J. Roy. Meteor. Soc.*, **104**: 331—350.

White, A.A., and Green J.A.S. (1982), A non-linear atmospheric long-wave model incorporating parameterizations of transient baroclinic eddies, *Quart. J. Roy. Meteor. Soc.*, **108**: 55—85.

Wiin-Nielsen, A., and Sela J. (1971), On the transport of quasi-geostrophic potential vorticity, *Mon. Wea. Rev.*, **99**: 447—459.

Wu Guoxiong (1983), The influence of large-scale orography upon the general circulation of the atmosphere, Ph.D. thesis, University of London, 203pp.

Wu Guoxiong (1984), The nonlinear response of the atmosphere to large-scale mechanical and thermal forcing, *J. Atmos. Sci.*, **41**: 2456—2476.

Wu Guoxiong and Dong Buwen (1986), Dynamic features of multiple equilibration in a forced dissipative system and energy levels of atmospheric motions, *Proceedings of International Summer Colloquium on Nonlinear Dynamics of the Atmosphere*, Beijing, Ed. Zeng, Science Press, pp. 79—98.

Wu Guoxiong and White A.A. (1986), A further study of the surface zonal flow predicted by an eddy flux parameterization scheme, *Quart. J. Roy. Meteor. Soc.*, **112**: 1041—1056.

Yao, M.S. (1980), Maintenance of quasi-stationary waves in a two-layer quasi-geostrophic spectral model with topography, *J. Atmos. Sci.*, **37**: 29—43.