A STUDY OF THE EFFECT OF GRAVITY WAVE BREAKING ON MIDDLE ATMOSPHERIC CIRCULATION*

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ABSTRACT

McFarline's gravity drag theory is simply reviewed, and it is indicated that the fault of McFarline's theory is that the effect of dissipation induced by gravity wave breaking on mean flow is not fully considered. Based on McFarline's theory, in this paper, the effect mentioned above is well considered. A new dissipation coefficient D is calculated, and a relatively complete parameterized scheme of the influence of gravity wave breaking on meanflow is put forward here. This is a better parameterized scheme than McFarline's if it is used in GCM.

Key words: middle atmosphere, wave breaking, dissipation effect, wave drag parameterization

I. INTRODUCTION

From the distribution of the temperature in the middle atmosphere (Fig. 1, see Murgetroyd 1969), which is obtained from observations, it is known that at about 50 km height in the atmosphere, there is an obvious reverse temperature gradient. It is clear that the departure from radiative equilibrium occurs in that area. In the special case, the temperature over the north pole can be 70—80°C higher than over the equator. What kind of cause may form this phenomenon? It may be inferred that besides photochemical reaction, there is the dynamic process which promotes the formation of the reverse temperature gradient.

This dynamic process is adiabatic ascending and adiabatic descending motion. If up to so obvious reverse temperature gradient, the vertical motion with 1.5 cm s^{-1} speed is necessary in dynamics. According to continuity demand, the meridional velocity component with about 5 m s^{-1} is needed . In terms of the function of Coriolis force torque, a surprised zonal wind acceleration can be approached up to 50 m s^{-1} d^{-1}. But, in fact, the atmospheric wind distribution obtained from observation is much smaller than the above calculation. Therefore, it must be considered that there is a much stronger zonal drag force. Otherwise, it is not satisfied in balancing the thermal and momentum budgets in the middle atmosphere.

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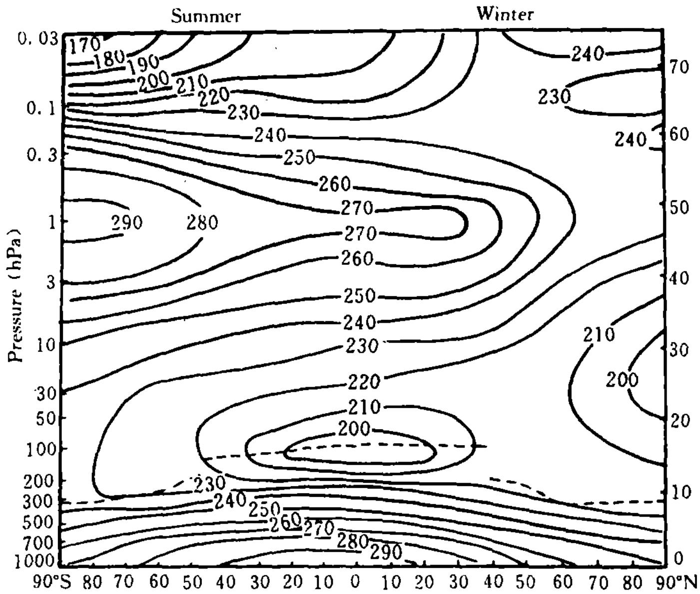

Fig. 1. The vertical distribution of atmospheric potential temperature (from Murgatroyd 1969).

This is why Rayleigh drag must be put in if modelling simulates the real wind distribution in the middle atmosphere. This drag effect may be dynamically referred to gravity wave or Rossby wave breaking.

II. McFARLANE's THEORY ABOUT GRAVITY WAVE DRAG

An important feature of the gravity wave propagation is that the momentum flux from source may be carried out by gravity wave up to middle atmosphere. In the gravity wave breaking region, the momentum flux carried out by gravity wave is dissipated, and it influences middle atmosphere in form of gravity wave drag. Many meteorologists have studied gravity wave phenomena in the past (e. g. Holton 1982; Lindzen 1981; Fritts 1984; Matsuno 1982). Readers can see many references involved if they are interested in this aspect. In this section McFarlane's gravity wave drag theory (McFarlane 1987) is introduced briefly, because currently McFarlane's parameterized scheme about gravity wave drag is adopted in some climate models, such as CCM2.

Suppose that terrain distribution is in the form

$$z = h\cos\mu x, \tag{1}$$

where h is the amplitude of the orographic perturbation. The important gravity wave properties may be summarized for a fixed wave source, and a steady gravity wave may be induced by the terrain. The basic equations can be written as

$$\bar{u}\frac{\partial u}{\partial x} + w\frac{\partial \bar{u}}{\partial z} = -\frac{\partial \pi}{\partial x}, \tag{2}$$

$$\frac{\partial \pi}{\partial z} = g\frac{\theta}{\bar{\theta}}, \tag{3}$$

$$\frac{\partial u}{\partial x} + \frac{1}{\bar{\rho}}\frac{\partial}{\partial z}(\bar{\rho}w) = 0, \tag{4}$$

$$\bar{u}\frac{\partial \theta}{\partial x} + w\frac{\partial \bar{\theta}}{\partial z} = 0, \tag{5}$$

where π is the Exner function, and $\pi = c_P P_0 \theta_0 (p/p_0)^K + gz$, the overbar $(-)$ indicates zonal average, and u, v, w are perturbations departed from average value, thus $\tilde{u} = \bar{u} + u$, $\tilde{\theta} = \bar{\theta} + \theta$.

In order to satisfy continuity equation, the following relations are defined as

$$w = \bar{u}\frac{\partial \psi}{\partial x}, \tag{6}$$

$$u = \frac{1}{\bar{\rho}}\frac{\partial}{\partial z}(\bar{\rho}\,\bar{u}\psi), \tag{7}$$

$$\theta = -\psi\frac{\partial \bar{\theta}}{\partial z}. \tag{8}$$

The bottom boundary condition is

$$\psi(x, 0) = h\cos\mu x. \tag{9}$$

Substituting Eqs. (6) — (8) into (2) — (5) obtains

$$\frac{\partial}{\partial z}\left[\frac{u^2}{\bar{\rho}}\frac{\partial}{\partial z}(\bar{\rho}\psi)\right] + N^2\psi = 0, \tag{10}$$

where $N^2 = (g/\bar{\theta})[\partial\bar{\theta}/\partial z]$. Here only wave phase changes with height. Because gravity wave is steady, and source of gravity wave is fixed, we have

$$\psi(z, x) = A(z)\cos\left[\mu x + \int_0^z \varphi(z')\mathrm{d}z'\right]. \tag{11}$$

Substituting (11) into (10), we get

$$\frac{N^2}{\bar{u}^2} - \varphi^2 + \mathrm{O}\left[\frac{1}{A}\frac{\mathrm{d}^2 A}{\mathrm{d}z^2}\right] = 0, \tag{12}$$

$$2\frac{\mathrm{d}A}{\mathrm{d}z} + A\left[\frac{1}{\varphi\bar{\rho}\,\bar{u}^2}\frac{\mathrm{d}}{\mathrm{d}z}(\varphi\bar{\rho}\,\bar{u}^2)\right] = 0. \tag{13}$$

From Eqs. (12) and (13), the leading-order approximation may be obtained

$$\varphi = \frac{N}{\bar{u}}, \tag{14}$$

$$A = h\left[\bar{\rho}(0)N(0)\frac{\bar{u}(0)}{\bar{\rho}N\bar{u}}\right]^{1/2}. \tag{15}$$

The mean vertical momentum flux for the wave may be defined as

$$\tau = \frac{1}{L}\int_{-\pi/L}^{\pi/L}\bar{\rho}uw\mathrm{d}x, \tag{16}$$

where the length scale L encompasses at least one horizontal wavelength of the wave.

Substituting Eq. (6) and (7) into (16), we obtain

$$\tau \approx -\frac{\mu h^2}{2}\bar{\rho}(0)N(0)\bar{u}(0). \tag{17}$$

This expression is independent of height, as expected, but is not valid in regions where wave dissipation processes are important, such as in saturation layer. When the wave propagation meets a saturation layer, $\partial\theta/\partial z \leqslant 0$ will occur and gravity wave will break.

Invoking Eq. (8), we have

$$\frac{\partial}{\partial z}(\bar{\theta} + \theta) = \frac{\partial\bar{\theta}}{\partial z}\left(1 - \frac{\partial\psi}{\partial z}\right) - \psi\left(\frac{\partial^2\bar{\theta}}{\partial z^2}\right) \leqslant 0. \tag{18}$$

The last term in this expression is usually negligible, (18) may be written as

$$\frac{\partial \bar{\theta}}{\partial z}\left(1-\frac{\partial \psi}{\partial z}\right) \leqslant 0. \tag{19}$$

Because the mean potential temperature increases with height, the wave breaking will occur in regions where the vertical gradient in the streamline displacement exceeds unity. It means $\partial \psi/\partial z > 1$, and this condition can be expressed as

$$F(z)\sin[\mu x+\int_0^z \varphi(z')\mathrm{d}z'] < 1, \tag{20}$$

where $F(z) = Nh/\bar{u}\{[\bar{\rho}(0)N(0)\bar{u}(0)]/[\bar{\rho}N\bar{u}]\}^{1/2}$ is the local Froude number.

In the saturation region, the amplitude of gravity wave is reduced because of wave breaking. If D indicates attenuation coefficient and it is a quantity to be determined, then we have

$$\psi_1 = \psi \exp[-\int_0^z D(z')\mathrm{d}z'], \tag{21}$$

$$\tau = \tau(0)\exp[-2\int_0^z D(z')\mathrm{d}z']. \tag{22}$$

Therefore, $F(z)$ may be written as

$$F(z)\exp[-\int_0^z D(z')\mathrm{d}z'] \leqslant 1. \tag{23}$$

In order to satisfy (23), $D(z)$ may be chosen as

$$D(z) = \frac{1}{F}\frac{\mathrm{d}F}{\mathrm{d}z}. \tag{24}$$

By utilizing (22) and (24), we have

$$\tau = \frac{\tau(0)}{F^2} = -\frac{1}{2}\left(\frac{\bar{\rho}\mu\bar{u}^3}{N}\right]. \tag{25}$$

Invoking the averaged momentum equation, the mean flow change in these circumstances is given by

$$\frac{\partial \bar{u}}{\partial t} = -\frac{1}{\bar{\rho}}\frac{\partial \tau}{\partial z} = -\frac{\mu}{2}\frac{\bar{u}^3}{N}\max\left[\frac{\mathrm{d}\ln F^2}{\mathrm{d}z},0\right]. \tag{26}$$

This is McFarlane's basic idea of gravity wave drag and the parameterized scheme about mean flow acceleration.

III. THE REVISION OF McFARLANE's THEORY ABOUT GRAVITY WAVE DRAG

In Section II, McFarlane's gravity wave drag theory is introduced briefly. According to his theory, before gravity wave breaking, the wave momentum flux carried by gravity wave propagation is independent of height under the condition of non-dissipation. When gravity wave propagation meets a saturation layer, the wave momentum flux will be reduced. Though McFarlane gave a reasonable parameterized scheme about the effect of wave momentum flux on mean flow acceleration, he did not consider the loss effect of wave momentum flux induced by wave breaking on mean flow acceleration, and the loss of wave momentum flux will produce dissipation.

The dissipation effect is also very important in the aspect of influence on mean flow acceleration.

From the viewpoint of momentum conservation, the wave momentum flux carried by gravity wave propagation is reduced as gravity wave goes up to saturation region. Where does the reduced part of the wave momentum go to? Whether or not does the reduced part of wave momentum also influence mean flow? It is worth while considering and studying the problem.

In this section, the target is made for compensating the fault of McFarlane's theory, and a revised parameterized scheme of gravity wave drag is pointed out.

In the wave breaking region, the dissipation induced by wave breaking occurs. Under the significance of differentiation, dissipative effect leads to full mixing. Therefore, mean flow in the saturation region may be considered as independence of z under the significance of differentiation. This is a better approximation. Because of full mixing assumption under the significance of differentiation, the air density variation is only a little bit with height. Therefore in the saturation region Eq. (10) may be aproximatively written as

$$\bar{u}^2 \frac{\partial^2 \psi}{\partial z^2} + N^2 \psi = 0. \tag{27}$$

Differentiating (27) with respect to x, and invoking (6), we have

$$W_{zz} + \frac{N^2}{\bar{u}^2} W = 0. \tag{28}$$

From (28), W may be expressed as

$$W = Ae^{i(kx+mz)}. \tag{29}$$

Substituting (29) into (28), and from knowledge that wave number must be positive, we have

$$m = \frac{N}{\bar{u}} = \varphi(z). \tag{30}$$

In the saturation region, the loss of wave momentum flux is transferred into frictional dissipation, and this dissipation may be written as

$$D \frac{\partial^2}{\partial z^2} \begin{Bmatrix} u \\ \theta \end{Bmatrix} = - m^2 D \begin{Bmatrix} u \\ \theta \end{Bmatrix}. \tag{31}$$

The thermodynamic equation may be written as

$$ik\bar{u}\theta + w\bar{\theta}_z = - m^2 D\theta. \tag{32}$$

Differentiating (32) with respect to Z, the leading-order approximation is

$$(ik\bar{u} + m^2 D)\theta_z + im\bar{\theta}_z w + w\bar{\theta}_{zz} = 0. \tag{33}$$

By utilizing (4) and (5), as a first-order approximation, we have

$$\theta_z = - \frac{mw\bar{\theta}_z}{k\bar{u}}. \tag{34}$$

Substituting (34) into (33) and letting their real part and imaginary part equalize respectively, we get

$$- D \frac{m^3 \bar{\theta}_z}{k\bar{u}} w + w\bar{\theta}_{zz} = 0, \tag{35}$$

$$- im\bar{\theta}_z w + im\bar{\theta}_z w = 0. \tag{36}$$

From (35), we obtain

$$D = \frac{k\,\bar{u}\,\bar{\theta}_{zz}}{m^3\,\bar{\theta}_z}. \tag{37}$$

Substituting (30) into (37), we can find

$$D = \frac{k\bar{u}^4 \bar{\theta}_{zz}}{N^3 \bar{\theta}_z}, \tag{38}$$

which is a parameterized expression of dissipative coefficient D. The variation of zonal mean velocity component induced by dissipation may be written as

$$\frac{\partial \tilde{u}}{\partial t} \approx D \frac{\partial^2 \tilde{u}}{\partial z^2}, \tag{39}$$

where $\tilde{u} = \bar{u} + u$.

According to the above assumptions, Expression (39) may be written as

$$\frac{\partial \tilde{u}}{\partial t} = D \frac{\partial^2 \bar{u}}{\partial z^2} + D \frac{\partial^2 u}{\partial z^2} = - m^2 D u. \tag{40}$$

In terms of the averaged difference between observational zonal wind and mean zonal wind, the following parameterized scheme

$$|u| \approx \frac{1}{3} |\tilde{u}| \tag{41}$$

is used. For west wind perturbation $u>0$, (40) may be written as

$$\frac{\partial \tilde{u}}{\partial t} = - m^2 D u = - \frac{m^2 D}{3} |\tilde{u}| = - \frac{N^2}{3\bar{u}^2} D \tilde{u}. \tag{42}$$

For east wind perturbation $u<0$, (40) may be written as

$$\frac{\partial \tilde{u}}{\partial t} = - m^2 D u = \frac{m^2 D}{3} |\tilde{u}| = - \frac{N^2}{3\bar{u}^2} D \tilde{u}. \tag{43}$$

For zonal averaged situation, we have

$$\frac{\partial \bar{u}}{\partial t} = - \frac{N^2}{3\bar{u}^2} D \bar{u}. \tag{44}$$

Finally, a complete parameterized scheme may be expressed as

$$\frac{\partial \bar{u}}{\partial t} = - \overline{\frac{1}{\rho} \frac{\partial \tau}{\partial z}} + D \frac{\partial^2 \bar{u}}{\partial z^2} = - \frac{k}{2} \frac{\bar{u}^3}{N} \max\left[\mathrm{d}\left(\frac{\ln F^2}{\mathrm{d}z} \right), 0 \right] - \frac{k \bar{u}^3 \theta_{zz}}{3N\theta_z}. \tag{45}$$

IV. CONCLUSION

The gravity wave breaking is one of important factors that impact the middle atmospheric circulation.

How gravity wave drag is parameterized is a hot research subject in the modern numerical modeling. The main problem which remains in McFarlane's theory is probed and solved in this paper, and a better parameterized scheme of gravity wave drag is given here for GCM use. But as a much better parameterized scheme, it is necessary to consider non-steady propagation of gravity wave and other factors. A relatively complete parameterized scheme will be given in another paper. Here only McFarlane's parameterized scheme of gravity wave drag is improved. A revised parameterized scheme has a simple form and may be used in GCM.

Finally, we must mention that because mathematic model is simple, the results obtained from the model still have some faults. It needs to be improved in the future.

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