A SIMPLE PROGNOSTIC CLOSURE ASSUMPTION TO DEEP CONVECTIVE PARAMETERIZATION: I

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ABSTRACT

In this work, the problem of dependency of the predicted rainfall upon the grid-size in mesoscale numerical weather prediction models is addressed. We argue that this problem is due to (i) the violation of the quasi-equilibrium assumption, which is underlying most existing convective parameterization schemes, and states that the convective activity may be considered in instantaneous equilibrium with the larger-scale forcing; and (ii) the violation of the hydrostatic approximation, made in most mesoscale models, which would induce too large-scale circulation in occurrence of strong convection. On the contrary, meso-β and meso-α scale models, i.e. models with horizontal grid size ranging from 10 to 100 km, have a capacity to resolve motions with characteristic scales close to the ones of the convective motions. We hypothesize that a possible way to eliminate this problem is (i) to take a prognostic approach to the parameterization of deep convection, whereby the quantities that describe the activity of convection are no longer diagnosed from the instantaneous value of the large-scale forcing, but predicted by time-dependent equations, that integrate the large-scale forcing over time; (ii) to introduce a mesoscale parameter which varies systematically with the grid size of the numerical model in order to damp large-scale circulation usually too induced when the grid size becomes smaller (from 100 km to 10 km). We propose an implementation of this idea in the frame of one existing scheme, already tested and used for a long time at the French Weather Service. The results of the test through one-dimensional experiments with the Phase III of GATE data are reported in this paper; and the ones on its implementation in the three-dimensional model with the OSCAR data will be reported in a companion paper.

Key words: prognostic clousure assumption, convection parameterization, 1D experiment, sensitivity experiment

I. INTRODUCTION

It is well known that cumulus convection is an important atmospheric phenomena. It plays a particular role in the vertical redistribution of heat and moisture in the atmosphere, thereby contributing to maintenance of the atmospheric general circulation. Because of the recognition of the importance of cumulus convection, numerous cumulus parameterization schemes have been proposed for numerical weather prediction (NWP) models, originating in the work of Smagorinsky (1956). A review of these schemes has been given by Frank (1983). Existing schemes may be classified in cumulus parameterization schemes for mesoscale numerical models (Kreitzberg and Perkey, 1976; Fritsch and Chappell, 1980; Frank and Cohen, 1987), and the ones for large-scale numerical models. The latter range historically from pumping schemes (Ooyama, 1963; 1969; Charney and Eliassen, 1964), through moist convective adjustment schemes (Manabe et al.,1965; Miyakoda, 1969; Kurihara, 1973), to penetrating convection schemes (Kuo, 1965; 1974; Anthes, 1977; Krishnamurti et al., 1983; Geleyn, 1985) and convective mass flux schemes (Arakawa and Schubert, 1974; Bougeault, 1985).

Among those, the schemes based on original idea by Kuo (1965; 1974,) and Arakawa and Schubert (1974, AS for short hereinafter) are the most widely used in most operational NWP models. The essential difference between these two schemes is their closure assumption. In first one, the theory of a partitioning of total large–scale supply of moisture is used, and so the amount of convection is related to the rate of moisture convergence by the environmental forcing. And in second one, it is made use of the relationship between the rate of destabilization by the large–scale environment and the convective activity. But, the conditional instability may previously be expected by either of them, in another word, the positive buoyancy is indispensable. In another important type of convective scheme, the dependence of convective available potential energy is closely related to the development of individual clouds for the closure assumption (Fritsch and Chappell, 1980, for example).

With the quick progress of super–computers, it becomes possible to utilize operational or research NWP models with smaller and smaller grid size and even variable grid size (e. g., the project of the French Weather Service: ARPEGE–Action de Recherche Petite Echelle et Grande Echelle–, Geleyn et al., 1988; Courtier and Geleyn, 1988). Of course, the smaller the grid size, the better the NWP models are capable of representing meteorological phenomena at smaller scales. However, when the grid size of an NWP model is decreased, a new specific problem arises; the predicted convective rainfall becomes very sensitive to the grid size. Moreover, the occurrence of anomalous grid point storms generally increases. This represents a great inconvenience for classical cumulus parameterization schemes now, and becomes a new problem in the theory of cumulus parameterization (Degardin and Imbart, 1987; Bougeault and Geleyn, 1989). How can one build a cumulus parameterization scheme allowing for a realistic prediction of convective rainfall independent of the grid size of the host model? One might argue that the host model should take care of this problem by itself, allowing for a progressive change from parameterized (so–called convective) to resolved (so–called grid scale) rainfall when the resolution is increased. However, in the current practice of most numerical models, it is known that the convective rainfall increases instead of decreases, when the grid size is reduced, because the model generates more active mesoscale circulation, and the large–scale precipitation is far from adjusting in such a way as to keep the total rainfall constant.

It is obvious that the problem becomes different for models that have a capacity to resolve the convective motions, i.e. non–hydrostatic models with a grid size of a few km. We are not addressing this problem here, but the problem of meso–β to meso–α scale, hydrostatic models, with a grid–size ranging from 10 km to 100 km, which Bougeault and Geleyn (1989) referred to as a "critical range". To our knowledge, until now, a scheme which is adaptable to an NWP model with a variable grid size does not exist, nor done the rationale behind such a scheme.

Since we want to build a scheme which works as well as for large and small grid sizes, it is interesting to start from the concepts in the existing schemes for the mesoscale and large–scale NWP models. At first, we recall two important concepts: the Convective Available Potential Energy (*CAPE*) and the Rate of Moisture Convergence (*RMC*). Usually, the *CAPE* is defined as an integral measure of the work of the buoyancy force from cumulus cloud bottom to its top:

$$CAPE = -\int_{P_b}^{P_t} g\frac{(T_{vc} - \overline{T}_v)}{\overline{T}_v} \cdot \frac{\mathrm{d}p}{\rho g}, \tag{1}$$

where T_{vc} denotes the virtual temperature in the cumulus clouds; $\overline{T}_v$ the virtual temperature in the environment; g the acceleration of gravity; P_b and P_t the pressure at cumulus bottom and top; ρ the atmospheric density.

On the other hand, one often defines the RMC as an integral measure of the rate of moisture convergence by the large-scale forcing and the diffusing processes:

$$RMC = -\int_{P_t}^{P_b} \left[\overline{\mathbf{V}} \cdot \nabla \overline{q} + \overline{\omega}\frac{\partial \overline{q}}{\partial p}\right] \cdot \frac{\mathrm{d}p}{g} + [F_q(P_b) - F_q(P_t)], \tag{2}$$

where F_q denotes the vertical moisture diffusion flux; $\overline{q}$ the specific humidity in the large scale environment; $\overline{\mathbf{V}}$ and $\overline{\omega}$ the horizontal and vertical velocities, respectively. All the schemes are more or less based on these two concepts ($CAPE$ and RMC), with some variations in their definitions.

If we compare the schemes for the large scale and mesoscale NWP models, we find that the most important difference is whether or not the idea of an instantaneous equilibrium between the cumulus convection and the environmental forcing is used. In order to fully address this problem, we have constructed a new scheme, based on a prognostic approach, which tries to render the results independent of the NWP model time step. And then, we systematically have tested the new scheme in one-dimensional (1D), two dimensional (2D), and found that the inclusion of the approach was not sufficient to reach our goal, although it already gave a large improvement over other schemes. We have therefore developed our work along another line, modifying the expression of the RMC, in order to include an explicit dependence upon the grid size of the NWP model.

We first describe the approach of the new cumulus parameterization scheme with a prognostic closure assumption in Section II. One-dimensional experiment results obtained with the new scheme are reported in Section III. We explore in Section IV the sensitivity of the new scheme. Finally, we give a conclusion in Section V.

II. PROGNOSTIC CUMULUS PARAMETERIZATION SCHEME

1. *Basic Idea*

In order to describe the physical basis of the idea of a prognostic closure assumption to cumulus parameterization, we would like to recall the discussion given by AS (1974). Imagine that one has an initial ideal atmosphere where $CAPE$ is zero, where there is no cumulus convection, no convective rainfall, but with an initial environmental forcing which can modify the vertical distribution of temperature and moisture (destabilization). After some time, through the environmental forcing, $CAPE$ will be accumulated in the atmosphere. As soon as one part of the accumulated $CAPE$ is released in form of the latent heat by any dynamic mechanism, the convection is triggered. It results in precipitations, and the convective response (or convective adjustment) to the environmental forcing is thus realized (warming and drying the large-scale environment). On one hant, the convective development will be toward equilibrating the atmosphere by consuming the accumulated $CAPE$ (stabilization); on the other hand, the environmental forcing will be toward increasing the $CAPE$ (destabilization). If the environmental forcing is stationary, it exists a possible moment at which the increase of the $CAPE$ by the environmental forcing can be equal to its decrease by the convection development. In other words, there is an equilibrium We call the time needed to reach the equilibrium state, the convective development time (or "adjustment time"). In reality, the environmental forcing changes always in time, and an exact equilibrium state will never be reached. The cumulus activity must depend not only on the environmental forcing at a given moment, but also on the past history of the environmental forcing and the cumulus activity. However, "this dependence should be significant only within the time scale of the adjustment time" (AS, 1974). According to their analysis, in a cumulus parameterization scheme for a large-scale model, the time scale of the environmental forcing ($>10^5$ s) is sufficiently longer than the adjustment time (10^3—10^4 s) for ignoring the influence of its time change. Therefore, the equilibrium between the cumulus clouds and the large-scale environmental forcing can be considered as almost instantaneous. This is the famous quasi-equilibrium assumption (AS, 1974).

However, in a cumulus parameterization scheme for a mesoscale model (for example 10 km $<\Delta x<$ 100 km), the convective adjustment processes no longer can be "filtered" in the time scale of environmental forcing, because of the time scale decrease of the phenomena represented by the dynamic model. On the contrary, they must seriously be taken into account. Some measures of the convective activity (for example, the convective mass flux or the fractional area of the grid box covered by cumulus clouds) must become prognostic variables. In this case, the convective activity no longer is in instantaneous equilibrium with the environmental forcing.

Such efforts to abandon the idea of an instantaneous equilibrium between the cumulus clouds and the environmental forcing have already been made in some past schemes for mesoscale models, in which one uses some time scales which usually are different from the time step of the host model for the feedback determining the final quantitative effects of the convection on its environment, either a smaller time step for the convection scheme (Kreitzberg and Perkey, 1976), a characteristic time scale based on horizontal advection (across a grid box) (Fritsch and Chappell, 1980), a time scaling factor that a parcel need it to rise from cloud base to cloud top or a simple method allowing to take into account some information on the past history of convective activity (Frank and Cohen, 1987). It may be physically absurd that the time scale used in a cumulus parameterization scheme is different from the time step of the host model. For example, taking a time step Δt ($=10$ min, for $\Delta x=50$ km) of the host model smaller than a characteristic time scale used in the scheme τ_c($=30$ min to 1 h). If one makes a prediction of one time step (10 min), one must "stop" the time change of the environmental forcing for about a half hour before the scheme gives a feedback on the environmental forcing. That is absolutely impossible in the real atmosphere. It may be true for a large-scale model because the quasi-equilibrium assumption is acceptable in this case. However, the schemes designed for large-scale NWP models are all based on the hypothesis of an "instantaneous" equilibrium, and by construction can not work at those scales. In any case, none of them contains a full prognostic variable for the representation of convective activity, that is that the cumulus activity generated in this way never really depends upon its past history. They are therefore, in our opinion, not sufficient to address the problem posed.

For simplicity, from now on, a cumulus parameterization scheme having a prognostic closure assumption for the representation of convective activity, is named a prognostic scheme; otherwise, it is named a diagnostic scheme. The "prognostic" means here on both count, at first, the variables for the representation of cumulus activity must explicitly depend on its past history; at second, the time scale used in a cumulus parameterization scheme must be the same as the time step of the host model in order to insure that the cumulus activity generated and the environmental forcing change at the same time. The scheme we want to build is a prognostic or quasi-prognostic scheme.

2. *Basic Equations*

The essential point of the cumulus parameterization problem is to determine the vertical heating and moistening amounts and distribution, along with convective rainfall. Hence, the thermodynamic equation and the moisture conservation equation usually are used as basic equations:

$$\frac{\partial s}{\partial t} + \nabla \cdot (\mathbf{V} \cdot s) + \frac{\partial \omega \cdot s}{\partial p} = C_p Q_r + LC, \tag{3}$$

$$\frac{\partial q}{\partial t} + \nabla \cdot (\mathbf{V} \cdot q) + \frac{\partial \omega \cdot q}{\partial p} = -C, \tag{4}$$

where $s = C_pT + gz$ represents the dry static energy; Q_r the radiation heating or cooling; C the net condensation rate and L the latent heat per unit mass of water vapour. If we average Eqs. (3) and (4) over the grid box with the assumption that this average verifies the Reynolds' axioms, we obtain with the usual notations for the large-scale effects of convection (Yanai et al., 1973; Ogura and Cho, 1973; Anthes, 1977)

$$Q_1 - \overline{Q}_r = \frac{1}{C_p}\left[L\overline{C} - \nabla \cdot \left(\overline{\mathbf{V}' \cdot s'}\right) - \frac{\partial \overline{\omega' \cdot s'}}{\partial p}\right], \tag{5}$$

$$-Q_2 = \frac{1}{C_p}\left[-\overline{C} - \nabla \cdot \left(\overline{\mathbf{V}' \cdot q'}\right) - \frac{\partial \overline{\omega' \cdot q'}}{\partial p}\right], \tag{6}$$

where Q_1 and Q_2 are the "apparent heat source" and "apparent moisture sink", expressed in K s^{-1}; the last two terms can be considered as the effects of the subgrid-size physical processes on the average value of temperature and moisture in the grid box. However, we will only consider the convective part in our case. In the same way, one could write two other equations for the effects of convection on momentum ("cumulus frictions"). Since the importance of these latter terms has been debated, and is still not clear. We will not discuss them for the sake of simplicity.

Now, suppose for $\Phi = s$ or q

$$\Phi = \overline{\Phi} + \Phi', \tag{7}$$

$$\overline{\Phi} = \sigma_c \cdot \Phi_c + (1 - \sigma_c) \cdot \Phi_e, \tag{8}$$

where $\overline{\Phi}$ represents the average value of s or q over the grid box; Φ' the fluctuation from the average value $\overline{\Phi}$; σ_c the fractional area of the grid box covered by cumulus clouds; the subscript "c" indicates the value within cumulus clouds; and the subscript "e" the value in the environment of cumulus clouds. Using Eqs. (7) and (8) we have

$$Q_1 - \overline{Q}_r = \frac{1}{C_p}\left[L\overline{C} - \frac{\partial \omega^* \cdot (s_c - \overline{s})}{\partial p}\right], \tag{9}$$

$$-Q_2 = \frac{1}{C_p}\left[-\overline{C} - \frac{\partial \omega^* \cdot (q_c - \overline{q})}{\partial p}\right], \tag{10}$$

where ω^* is equal to $\sigma_c \cdot (\omega_c - \omega_e)$; the horizontal advection terms $[\nabla \cdot (\overline{\mathbf{V}' \cdot \Phi'})]$ were neglected for the sake of simplicity. If the hypotheses ($\sigma_c \ll 1$) and ($\omega_c \gg \overline{\omega}$) are introduced (as would be the case for any large grid box), then $-\omega^*$ is nearly equal to the convective mass flux usually defined as M_c ($= -\sigma_c \cdot \omega_c$).

In order to establish our new prognostic scheme, we start from the formulation of Bougeault (1985, B85 in the following), because of its simplicity and its efficiency demonstrated by its use since 1985 in the large-scale operational NWP model EMERAUDE (a French spectrum large-scale operational model) of the French Weather Service. This scheme is based on a drastic simplification of the previous equations, reading

$$Q_1 = \frac{1}{C_p}\left[-\omega^* \frac{\partial \bar{s}}{\partial p} + K(s_c - \bar{s})\right], \tag{11}$$

$$-Q_2 = \frac{L}{C_p}\left[-\omega^* \frac{\partial \bar{q}}{\partial p} + K(q_c - \bar{q})\right], \tag{12}$$

where K is an inverse time scale for recycling cloud water into the environment. The B85 scheme postulates a simple profile for the convective mass flux ω^*, in which both *RMC* and *CAPE* (in an approximate local form) were made use of. And K is determined by imposing that Eqs. (11) and (12) conserve the total moist static energy of the convective column. The B85 scheme is a KUO-type scheme (1965; 1974) improved by the approach of AS scheme (1974), since it uses both approaches of the convective mass flux terms and the relaxation terms of temperature and moisture.

For our new prognostic scheme, we want to generalize the B85 scheme by determining the convective mass flux in another way. We firstly decompose the convective mass flux in two parts: the cloud-scale vertical velocity ω_c^*, and the fractional area of the grid box covered by cumulus clouds (more precisely by convective updrafts), that is:

$$\omega^* = \alpha^* \cdot \omega_c^*. \tag{13}$$

Then, we find two equations that allow us to determine these quantities. Of course, the ω_c^* and α_c^* defined here are different from those traditionally defined ω_c and α_c. The formers only represent, respectively, the vertical velocity of convective updrafts (it is therefore always negative), the fractional area of the grid box covered by convective updrafts (α^* is therefore always smaller than or equal to α_c).

3. *Cloud-Scale Vertical Velocity* ω_c^*

The vertical acceleration of an isolated cloud parcel (without pressure perturbation and mixing with the surrounding air), can be classically given by the following equation of vertical motion in pressure ordinate:

$$-\frac{1}{\rho_c g} \cdot \frac{d\omega_c}{dt} = g\frac{(T_{vc} - \bar{T}_v)}{\bar{T}_v}, \tag{14}$$

where the hydrostatic assumption was used for the environment; ρ_c and $\bar{\rho}$ represent the density of the cloud air and the environment. Then, we make for Eq. (14) same transformation as Simpson and Wiggert (1969), and Simpson (1971) in keeping the local time change and the vertical advection terms of ω_c

$$\frac{\partial\omega_c}{\partial t} = -\beta\frac{(T_{vc} - \bar{T}_v)}{(1+\gamma')} + \lambda_e\omega_c^2 + K_d\omega_c^2, \tag{15}$$

with

$$\beta = \frac{\rho_c g^2}{T_v} \qquad \text{and} \qquad \lambda_e = -\frac{2}{M_c} \cdot \frac{\partial M_c}{\partial P},$$

where we have used the relation: $M_c = -\sigma_c \omega_c$; $\gamma' = 0.5$ is a so-call virtual mass parameter; the second term on the right side of Eq. (15) is an entrainment term; $\lambda_e / 2$ is a rate of entrainment; the third is a drag term, with K_d usually constant. For a convective updrafts, the last two terms on the right side can be taken as an only term-dissipating term, since it is known that the entrainment and drag processes are dissipating. Noting that the last two terms both contain a squared cloud-scale velocity ω_c^2, this dissipating terms may be considered as a structure function of $\omega_c^* \nabla^2$ multiplied by a coefficient, which could be related to the convective updrafts' intensity variable (for example, the total thickness of convective layer). So, in order to further simplify the quasi-full equation (15), we deduce the following approximation for determining the cloud-scale velocity:

$$\frac{\partial \omega_c^*}{\partial t} \simeq \frac{\omega_c^{*2}}{\Delta P_t} \cdot C_f - \frac{(T_{vc} - \overline{T}_v)}{(1+\gamma')} \cdot \beta, \tag{16}$$

where $\Delta P_t = P_b - P_t$ denotes the total thickness of the convective layer; the second term on the right side of Eq. (16) is the term of buoyancy, whose integral over the vertical can be considered as the *CAPE*. The first term is a term representing globally the effects of smaller convective scale processes, with C_f an adjustable dissipating coefficient, whose optimum value is found empirically around 50. Eq. (16) means that the time change of the cloud-scale velocity results from a balance between the buoyancy and the dissipation. The first term on the right is always positive, so dissipating; it destroys the acceleration of convective updrafts (since ω_c^* is negative). The buoyancy term (second term), on the contrary, is always negative within cumulus clouds, so it contributes to development of the updrafts. Moreover, the greater the convective thickness, the less the dissipation.

4. *Fractional Area of the Grid Box Covered by Updrafts*

It is known that most classical diagnostic schemes do not attempt to represent the storage of moisture within the clouds. However, meteorological observations indicate that the life time of an individual cumulonimbus is about 1 hour (of which one half hour for its development, and another half hour for its dissipation, assuming that it is symmetrical in time). For a large-scale NWP model ($\Delta t > 20$ min, for $\Delta x > 100$), it is well acceptable to suppress the storage term and the convective vertical acceleration term within the cumulus clouds. But, for a mesoscale numerical model, it is not reasonable to neglect them, because the time step of the model is much smaller!

Hence, in order to calculate the fractional area of the grid box covered by updrafts α^*, consistently with the hypothesis that the convective updrafts are not stationary equilibrium with the large-scale environment (Eq. (16)), we proposed a prognostic closure assumption, starting from the diagnostic closure assumption of the B85 scheme:

$$\frac{\partial \alpha^*}{\partial t} \cdot \int_{P_t}^{P_b} (h_c - \overline{h}) \frac{dp}{g} = L \int_{P_t}^{P_b} \omega^* \frac{\partial \overline{q}}{\partial p} \cdot \frac{dp}{g} + L \cdot RMC. \tag{17}$$

The term on the left hand side can be considered as the storage rate of moist static energy integrated over the convective column; the second term on the right is the total available latent energy by the environmental forcing; the first term represents the instantaneous destruction of this energy by the convective development. Eq. (17) therefore states that the latent energy ($L \cdot RMC$) supplied by the large-scale forcing is primarily used to fuel the convective development, while the remainder is stored within cumulus clouds in form not only of latent energy, but also of sensible energy. In fact, in the real atmosphere, it could be difficult to imagine that the latent energy stored within a cumulus cloud can keep an only form of latent energy, in stead of both latent energy and sensible energy.

It is important to realize that Eq. (17) does not embody the conservation of moist static energy by the convective processes. This property is still insured by computation of the K factor. Eq. (17) is the closure equation of the scheme. As such, it still constrains a large degree of arbitrariness.

It is obvious from Eqs. (16) and (17) that the physical mechanism described by the new prognostic scheme represents an interaction between the cumulus cloud ensemble and the large-scale environment, since it uses both the $CAPE$ in Eq. (16) and the RMC in Eq. (17). Moreover, in the real atmosphere, there is sometimes some violent convective processes developing in a very small area, but with a strong vertical velocity (Lemone and Zipser, 1980). This phenomenona can also be represented by our new prognostic scheme. Indeed, considering Eqs. (16) and (17): for a RMC at a given moment, the more the buoyant work that is done, the stronger the cloud-scale velocity ω_c^* is, and the less important the convective fractional area α^* will be.

Except for the different determination of the convective mass flux ω^*, we keep all of the others principles, equations and algorithms of the B85 scheme in our new prognostic scheme. If we suppose that the convective updrafts are stationary ($\partial \omega_c^* / \partial t = 0$), that there is no storage in the closure ($\partial \alpha^* / \partial t = 0$) and that ($T_{vc} - \overline{T}_v$) is replaced by $[(h_c - \overline{h}) / C_p]$ in Eq. (16), Eqs. (16) and (17) automatically revert to ones of the B85 scheme (Eqs. (9) and (13), Bougeault, 1985) This is an important property of the new scheme, and as a consequence, we expect to get the results similar to those with the B85 scheme for large grid size and large time step. However, in our new prognostic scheme, the parameter α^* is a dimensionless coefficient. It represents the fractional area of the grid box covered by updrafts, whereas the α of the B85 scheme has no clear physical meaning. Another advantage of the new formulation is that ω_c^* can be compared to the observations of cloud-scale velocity.

5. *Some New Definitions of CAPE and RMC*

At the beginning, we have recalled the most widely used definitions of $CAPE$ and RMC (see Eqs. (1) and (2)). In our formulation of the prognostic scheme, the $CAPE$ is included in the term of buoyancy (see Eq. (16)) and the RMC in the term of total available latent energy by the environmental forcing (see Eq. (17)). During the development of this work, we have introduced some new definitions for these quantities.

Concerning the $CAPE$, we used a cloud profile T_c based on the classical moist adiabatic as in B85, but including liquid water remaining in suspension inside the clouds:

$$T_{vc} = T_c \left[1 + q_c \left(\frac{R_v - R_a}{R_a}\right)\right] - \frac{L}{C_p} q_l, \tag{18}$$

where q_c stands for the saturation value of the specific humidity at T_c inside the cumulus clouds; and q_l the liquid water specific humidity inside the clouds. This was computed in a simple way following a original method of AS (1974). The cloud profiles were thus diagnostically determined (from this point, strictly speaking, our new scheme is not a fully-prognostic scheme).

For the *RMC*, there also exist several approaches in the literature. Krishnamurti et al. (1980), and Kuo and Anthes (1984) have noted that convective rainfalls are often better correlated with the large-scale vertical moisture advection than the total moisture source, as defined in Eq. (2). However, when this term is retained alone, one usually underestimates the convective rainfall by as much as 20%. This has led Krishnamurti et al. (1983) to increase the vertical moisture advection by a numerical factor which is seen as a mesoscale dynamic parameter. We have tried a similar approach, based upon a somewhat different background, for a new closure assumption of the scheme:

$$RMC = -C_\beta \cdot \int_{P_t}^{P_b} \bar{\omega} \frac{\partial \bar{q}}{\partial p} \cdot \frac{dp}{g}, \tag{19}$$

where C_β is a parameter including an explicit dependence upon the grid-size. After some tests, we found that a nonlinear dependence (square grid-size) gave better results, which will be reported in the next paper. We found in essence the same justification as Krishnamurti et al. (1983) for this coefficient: it accounts for the existence of mesoscale circulation inside the grid box where the convection is parameterized. The larger the grid box, the larger the C_β must be, to take into account the increase of moisture available through these mesoscale circulations. These modifications of the closure turned out to one of the major developments of our work here. It will be termed in the following as the new closure assumption.

6. *Numerical Implementation of the Scheme*

In order to prevent the numerical instability which could arise from the prognostic equations (16) and (17), in the NWP model, we prefer to use an implicit time discretization scheme for calculating $(\omega_c^*)_t$ and $(\alpha^*)_t$. This reads

$$(\omega_c^*)_t = \frac{\frac{\Delta P_t}{\Delta t} - \sqrt{\left(\frac{\Delta P_t}{\Delta t}\right)^2 - 4C_f\left[\frac{\Delta P_t}{\Delta t}(\omega_c^*)_{t-\Delta t} - \frac{(T_{vc} - \bar{T}_v)}{(1+\gamma')}\beta \Delta P_t\right]}}{2C_f}, \tag{20}$$

$$(\alpha^*)_t = \frac{(\alpha^*)_{t-\Delta t} \int_{P_t}^{P_b} (h_c - \bar{h}) \frac{dp}{g} + L \cdot RMC \cdot \Delta t}{\int_{P_t}^{P_b} (h_c - \bar{h}) \frac{dp}{g} - L \int_{P_t}^{P_b} (\omega_c^*)_t \frac{\partial \bar{q}}{\partial p} \cdot \frac{dp}{g} \Delta t}, \tag{21}$$

where we have neglected the positive solution for Eq. (16), because we are interested only in the convective updrafts; Δt is the time step which is identical to that of the host model. The other algorithms are identical to those described in B85, except for the determination of the cloud profile.

In order to eliminate the problem that $\alpha^* > 1$, when the vertical integral of $(h_c - \bar{h})$ (which is the denominator of Eq. (21), and usually important) takes a small value, which corresponds to near-neutrality, we have imposed a threshold value on $(\omega_c)_t$. This means whenever $\| (\omega_c^*)_t \|$ computed is smaller than $\| (\omega_c^*)_t \|_{\min}$, it is assumed that there is no determination in Eq. (21), and so there is no convective mass flux. We therefore define the area affected by deep convection by the condition $\| (\omega_c^*)_t \| > \| (\omega_c^*)_t \|_{\min}$. However, when the condition is not fulfilled, $\omega_c^*(t)$ is not set to zero, so that the conditions favourable to deep convection can continue to develop in the next time step. The value of the threshold was taken as $\| (\omega_c)_t \|_{\min.} = 0.1$ m / s. We have verified that it is very efficient in avoiding the case where $\alpha^* > 1$ and that the results are only slightly modified by a change of this value in the interval 0.01—0.5 m / s.

In addition, for starting the computation, at the first time step, we assume

$$(\omega_c^*)_{t=0} \equiv 0, \tag{22}$$

$$(\alpha^*)_{t=0} \equiv 0, \tag{23}$$

since there is no way to know the exact initial state of the updrafts.

III. 1D EXPERIMENTS ON THE GATE DATASET

1. *Semi–Prognostic Model and the Dataset*

In order to test a cumulus parameterization scheme, the semi–prognostic 1D method (Lord, 1982; Krishnamurti et al., 1983) is generally used with a stationary large–scale foring because it allows one to "isolate" the potential problems of the scheme under study in a simplest way. This means that it allows one to avoid the confusion with any possible modeling errors other than those caused by the scheme itself, such as the errors caused by the other subgrid–scale physical process parameterization schemes, those by the unmerical model itself, etc. When testing any diagnostic scheme, it is a common practice to advance by one time step only with the semi–prognostic method. However, this is no longer suitable to test this prognostic scheme, since we assume $(\omega_c^*)_{t=0} = 0$ and $(\alpha^*)_{t=0} = 0$ as the initial conditions for starting the computation. In this case, what is important is the simulation results after setting up the convective equilibrium, not those ones at the first time step. Thus, in order to verify the results of one–dimensional experiments obtained by the new prognostic scheme, we will use the semi–prognostic method to advance by several time steps, until a stationary result is in occurrence. Furthermore, this will allow one to study the convection development time (or convective adjustment time) with the prognostic scheme, under a stationary large–scale forcing.

We have used for the present study the same 1D semi–prognostic model as in B85, and we refer reader to that paper for details. Prognostic equations are solved for the temperature and humidity profiles, with the vertical pressure velocity, the horizontal advection, and the radiative forcing specified from the observations, as analysed in the dataset produced by Esbensen et al. (1982). Minor differences with B85 were introduced: the vertical distribution of the computation levels is not the same. We use for present study 15 levels linearly distributed in pressure. As a consequence, the lowest model level is not at the same height above sea level, and the roughness length, used in computation of the surface fluxes of energy, has been adjusted to 3 mm (instead of 5 mm), in order to keep a correct average value of the surface fluxes.

The dataset of Esbensen et al. (1982) contains 161 independent specifications of the initial profiles, and of the large-scale forcing, valid every 3 hours from 30 August to 19 September 1974 (GATE Phase III). For each of these 161 profiles, the 1D model is initialized, and run for more than one hour, until a stationary state is reached (nearly no more change with time for Q_1, Q_2 and ω^* under the stationary large-scale forcing). In the following, we will mostly present the results averaged over the 161 individual, forecasts, in order to get insight into the performance of the scheme on a large number of independent situations. We will also present some figures where the abscissa displays the 161 individual forecasts at stationary, and the ordinate displays the depth of the atmosphere, for getting insight into the day-to-day variations of the convective patterns.

2. *Reference Experiment*

We will first describe extensively the results of a reference experiment, carried out with the prognostic scheme, the total available moisture *RMC* as defined by Eq. (2), and $C_f = 50$ in Eq. (16). We explore the scheme results after 72 min, i.e. when a perfectly stationary state is reached. Figs. 1 and 2 show the day-to-day evolution of the apparent heat source and apparent moisture sink due to convection. The observations are given in the upper panels, and the scheme results in the lower ones. Similar pictures have been published by many authors, and it is hardly necessary to describe them. Six intense events, occurred during the period, and were associated with easterly waves propagating over the area. They are clearly visible. They correspond to large values of Q_1 and Q_2 that are fairly well predicted. Similarly, the prediction of rainfall rates (Fig. 3) is acceptable. Moreover, taking a time average of observed and predicted Q_1 and Q_2 reveals no systematic errors in the heating and drying profiles (Fig. 5). Finally, it is clear from the absolute mean errors on Q_1, Q_2 and rainfall rate, shown in Table 1, that the prognostic scheme results are of comparable quality to those of the B85 scheme (see also that paper). This was to be expected, since the prognostic scheme has been constructed in order to be consistent with B85 at stationary.

The behaviour of the new prognostic variables α^* and ω_c^* can be investigated from their day-to-day variations, which are respectively shown in Figs. 3 and 4. The magnitudes of α^* clearly follow the convective activity, reaching 1% to 1.5% during the most active events. Recalling that α^* is designed as a measure of the fractional area of the grid box covered only by active updrafts (not total active clouds), this order of magnitude seems reasonable. On the other hand, we note only small variations in the vertical profile of ω_c^* from one day to another, which is consistent with the fact that only small variations of the *CAPE* are observed in the dataset. The maximum vertical velocity is generally of the order of 5 m / s. It is remarkable that Zipser and Lemone (1980) have obtained experimental values during GATE Phase III for the fractional area of the grid box covered by updrafts and for the vertical velocity inside clouds, that are well comparable with the values predicted here. We therefore claim that our internal variables do represent physical quantities which are meaningful.

We have run the reference experiment up to 72 min with 5 different values of the time step,

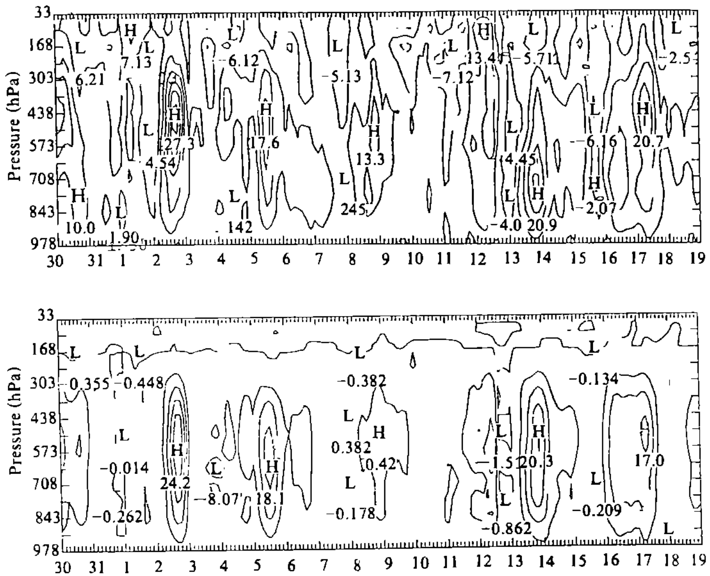


Fig. 1. Vertical cross-section of the apparent heat source Q_1-Q_r. Abscissa denotes dates during Phase III of GATE, from 30 August to 19 September, 1974. Units are K / d, and the interval between two isolines is 5K / d. Top panel: observations from Esbensen et al., 1982; Bottom panel: the predictions of the reference 1D experiment after 72 min (see text).

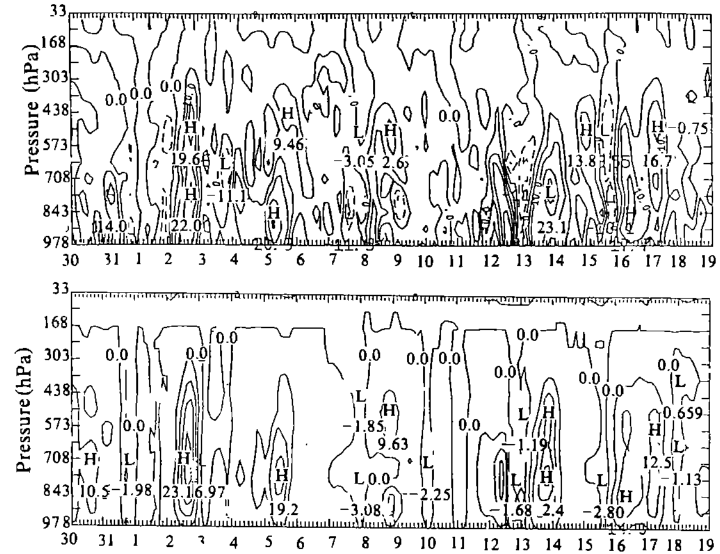


Fig. 2. As in Fig. 1, but for the apparent moisture sink Q_2, in K / d.

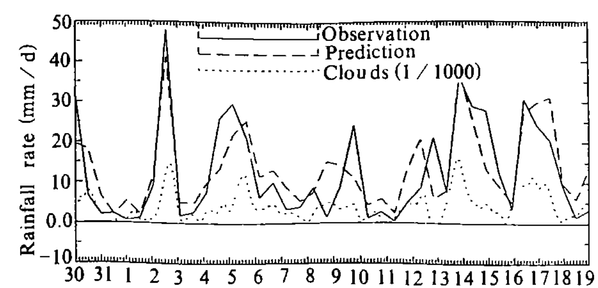
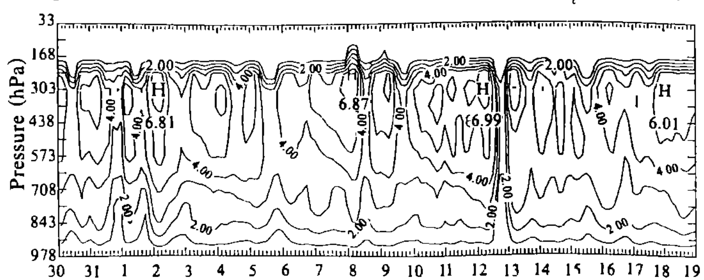


Fig. 3. Comparison between observed (solid line) and predicted (dashed line) rainfall rates for the reference experiment, and predicted fractional area of the grid box covered by convective updrafts (short-dashed line), in 1 / 1000.

Fig. 4. Vertical cross-section of the cloud-scale velocity ω_c^* (in m / s).



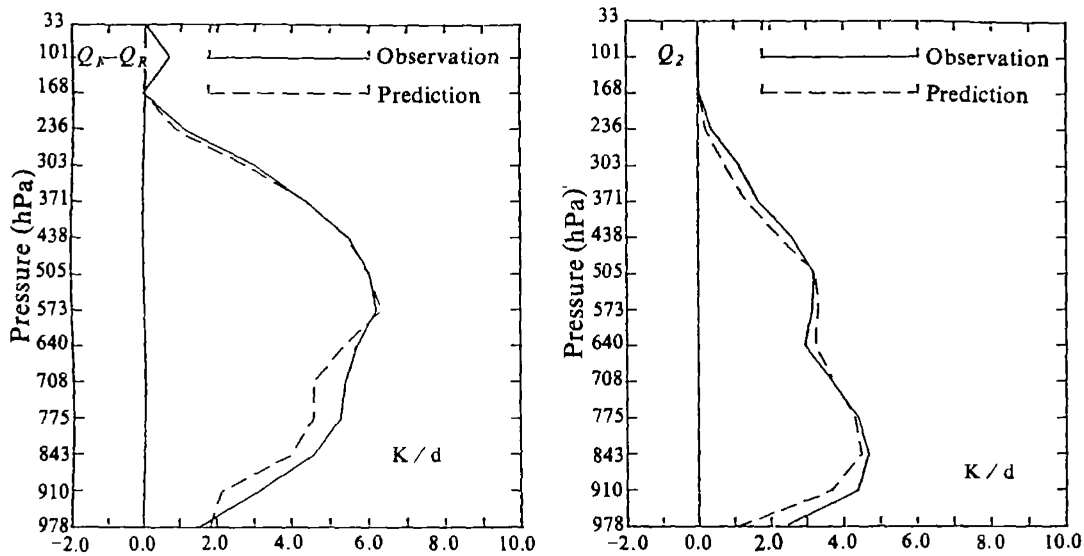


Fig. 5. 20-day (161 profiles) average of the observed and predicted (reference experiment) apparent heat source (left) and moisture sink (right).

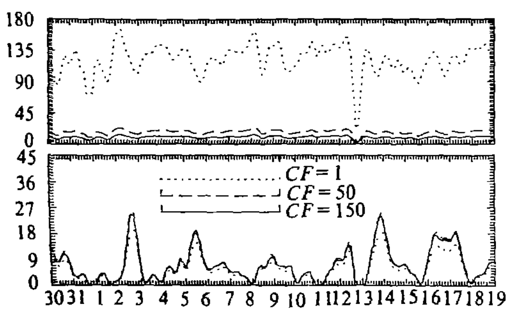


Fig. 6. Evolution of the cloud-scale velocity ω_c^* (top) and of the convective mass flux ω^* (bottom) for the sensitivity to C_f experiments. Both quantities are averaged over the vertical. Units are Pa / s on top panel, and 0.01 Pa / s on bottom.

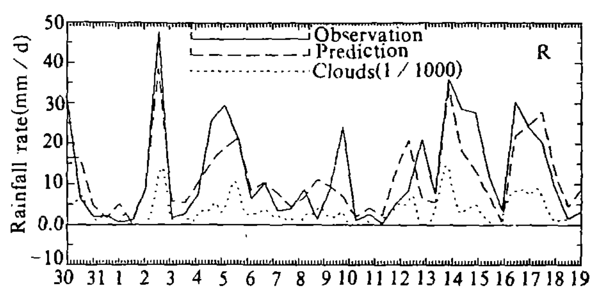


Fig. 7. As in Fig. 3, but for the experiment V_1 (see text).

covering the range used in mesoscale to large-scale numerical models (60s to 720s). We found that the stationary state reached after 72 min does not depend upon the time step. More important, however, the convergence speed is also essentially independent of the time step (figure not showed here, ref. Chen, 1989).

In order to study more quantitatively this problem, we have defined the response time t_r of the scheme for a given time step as the first time when the condition

$$\left\| \frac{\tilde{\Phi}(t+\Delta t)-\tilde{\Phi}(t)}{\Delta t} \right\| \leqslant \varepsilon_{\Phi} \tag{24}$$

is verified, with

$$\tilde{\Phi}(t)=\int_{P_1}^{P_{NK}}\left\{\frac{1}{161}\sum_{j=1}^{161}[\Phi(t,p)]_j\right\}\mathrm{d}p \tag{25}$$

and $\ni_\Phi$ being a small controlling value. This response time has been computed for several variables ($\Phi=\omega_c^*$, Q_1 and Q_2), and yielded very comparable values, that are shown in Fig. 6. It can be seen that the response time is indeed only slightly dependent upon the time step. Moreover, its value (25—30 min) is in fair agreement with the characteristic time scale for the development of a convective cloud. In order to compare this result with diagnostic schemes, we must recall that in such schemes, because of the quasi-equilibrium assumption, the response time is constrained to be equal to the time step of the host model. The corresponding curve has been plotted in Fig. 6. The picture therefore clearly demonstrates the superiority of the prognostic approach, which insures a realistic response times, nearly independent of the model time step. The same conclusion has be found for the other different prognostic schemes (see below). Based on these results, we hope to have solved one of the problems leading to the no-proper behaviour of most convection schemes in models with small grid sizes, and therefore small time steps.

IV. SENSITIVITY EXPERIMENTS

In order to investigate to what extent these results might depend upon minor details of the formulation, we have made several sensitivity experiments.

1. *Sensitivity to the C_f Coefficient*

The dissipation coefficient C_f is one of the main unknowns introduced by the new formulation (Eq. (16)). It is therefore important to explore the dependence of the results upon its value. We have run two other experiments with the prognostic scheme (*RMC* as defined by Eq. (2)), taking $C_f=1$ and $C_f=150$, and compared the results with the reference experiment ($C_f=50$). The main dependence found is of course on ω_c^* . The depth-averaged values of ω_c^* for the three cases are shown in Fig. 6: the smaller C_f , the larger ω_c^* . We may also note that for $C_f=1$, the time variations of ω_c^* is much larger, indicating a larger dependence upon the *CAPE*. However, it is remarkable that the changes in ω_c^* are almost totally compensated by the changes in α^*. As a consequence, the convective mass flux ω^* depends only weakly upon the C_f value. This is a most interesting result, since it means that a possible error on the optimum value of C_f will not induce a large error on the large-scale effects of convection. In other words, if we forget about the internal variables, the prognostic approach has no more degrees of freedom than the diagnostic scheme.

2. *Experiments with the New Closure Assumption*

It is of interest here to discuss some of our results with the new closure assumption introduced in Section II-5 (Eq.(19)), since it was used in our 3D experiments (see next paper). We have run two experiments with the prognostic scheme, $C_f=50$, but *RMC* as defined by Eq. (19) instead of Eq. (2): Experiment V_1 with $C_\beta=1$, experiment V_2 with $C_\beta=1.15$. The results

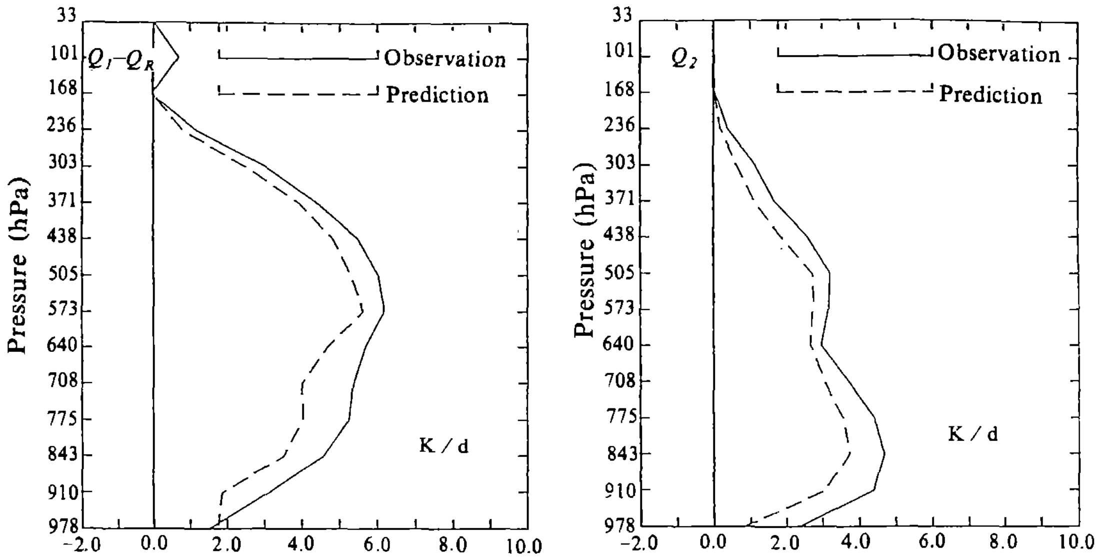


Fig. 8. As in Fig. 5, but for the experiment V_1 (see text).

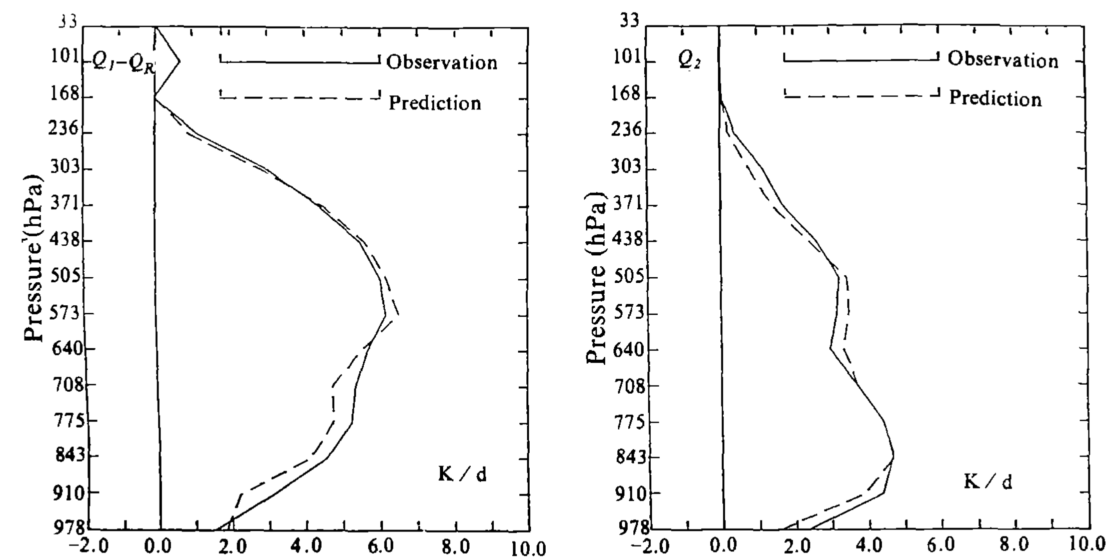


Fig. 9. As in Fig. 5, but for the experiment V_1 (see text).

are reported in Figs. 9 to 10, and in Table 1. In experiment V_1, the convective rainfall, the heating, and the drying are clearly underestimated (Figs.7 and 8), as compared to the observations and to the reference experiments. This is because its mean value has been significantly decreased by the removal of the moisture diffusion flux from the *RMC*. This problem is resolved in experiments V_2 (Figs.9 and 10), where the introduction of the larger value of C_β produces the correct averaged amount of the parameterized convection. It is interesting to note that the absolute mean errors (Table 1) are not larger for experiment V_2. In fact, they are even smaller than for the reference case and the more rainfall could be produced for the convective situation on 2 September 1974 (Fig.10), but such a small difference is probably not significant. We conclude that, the new closure assumption (Eq. (19)) is equally acceptable as the traditional form

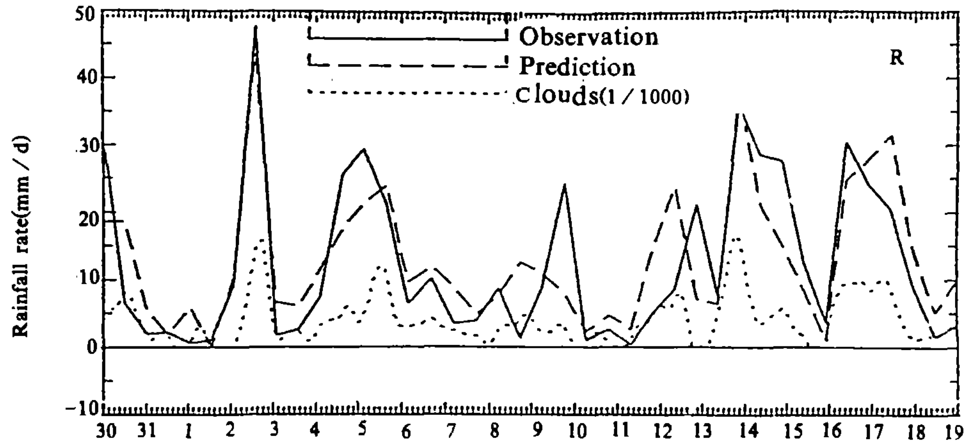


Fig. 10. As in Fig. 3, but for the experiment V_2 (see text).

Eq.(2). Given the degree of arbitrariness of any closure assumption, this opens the way for numerical experimentation with Eq. (19), where the C_β coefficient would be varied as a function of the grid-size of the host model, in such a way as to cancel the dependence of the convective activity from the grid-size. These experiment results will be reported in the next paper.

Table 1. Summary of Absolute Errors on the Whole GATE Phase III Dataset for All 1D Experiments

	Prognostic schemes					B85 scheme
	$C_f=150$	$C_f=50$	$C_f=1$	$V_1=1$	$V_2=1$	
E_{rain}(mm / d)	5.54	5.70	5.76	5.43	5.65	5.91
E_{Q1}(K / d)	1.99	1.99	1.99	2.02	1.97	2.06
E_{Q2}(K / d)	2.13	2.13	2.13	2.17	2.15	2.18

V. CONCLUSION

The purpose of this study is to define a new convective parameterization scheme, that would have a capacity to work as well as in NWP models with grid sizes from 10 km to 100 km. We have taken a gradual approach towards this goal, and first found that it would be useful to make the prognostic scheme in order to eliminate the dependence of the host model time steps. Indeed, diagnostic schemes are based upon the quasi-equilibrium assumption, that states that the time scale for adjustment of the convective cloud activity to the large-scale forcing is much smaller than the characteristic time scale of the large-scale flow variations. This assumption is violated in models with small grid size and small time steps, that have a capacity to resolve motions with characteristic scales close to the ones of convective scales, but not the convective scales themselves. We have hypothesized that one way to solve this problem is to introduce a prognostic closure assumption to the parameterization of convection, whereby the quantities that describe the convective clouds are no longer diagnosed from the instantaneous value of the large-scale forcing, but predicted by time-dependent equation, that integrate the large-scale forcing over time.

We have proposed in this paper one possible realization of this idea, based on the original scheme of Bougeault (1985). The convective mass flux, which is the central variable of the scheme, is split into two variables: the cloud scale vertical velocity, and the fractional area of the grid box covered by updrafts. Prognostic equations are written in a simple way for these two new equations, with the constraint to be consistent with the former formulation at stationary state. Experiment in 1D semi-prognostic model with the GATE dataset has demonstrated the interest and the feasibility of the approach. The results for the large-scale effects of convection (rainfall, heating and drying) are of comparable quality to those of the other known schemes. The new variables introduced take values in agreement with observations of related quantities available in the literature. The characteristic response time of the scheme is independent of the numerical time steps, and its values (25 to 30 min) are comparable to the characteristic time scale for the development of a convective cloud.

We have introduced an another new closure assumption, based on a different formulation of the *RMC*. The results of the 1D semi-prognostic test of this new closure assumption were equally acceptable, with regard to the GATE Phase III dataset.

Finally, we would like to mention that the concept of a prognostic scheme may probably be introduced in most existing parameterizations of convection, since it is based on general consideration, and not to on the details of the formulation of any particular scheme. Our sensitivity experiments showed that the details in the formulation were totally transparent to the modifications of the code to the results of the prognostic version. This may encourage our colleagues working in the field of convection parameterization to consider similar approach, without having to change the formulation in a radical way.

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